The Association
of
Engineering and Shipbuilding
Draughtsmen.

GEAR MANUFACTURE.

BOOK I.

Types of Gears.
Form of Tooth.
Correction of Spur Gears.

By F. WHITELEY.

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BOOK I.

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GEAR MANUFACTURE.

By F. Whiteley.

CHAPTER 1.

TYPES OF GEARS.

There are many different types of gears, and the classification or identification depends largely on how the gears are being used.

For example, when two spiral or skew gears are being used to connect two shafts not running parallel with each other they are termed spiral gears, but should the shafts be parallel with each other, similar gears used for connecting the two shafts are named helical gears.

The more important of the existing types of gears will therefore be described to assist in the matter of classification, and in each case, the chief characteristics required for the detail design of gears having standard proportions are also included.

Spur Gears. Fig. 1.

This type of gear is most commonly used to connect two parallel shafts, and which then rotate in opposite directions.

The teeth are parallel to the axis of the gear, and are identical in profile from one side of the face to the other.

They are subjected to no axial force due to tooth load, and are therefore used as sliding gears for change speed mechanisms in gearboxes.

The spur pinion (*i.e.*, the gear having the smaller number of teeth in a pair of mating gears) is also used to mesh with an internal spur gear.

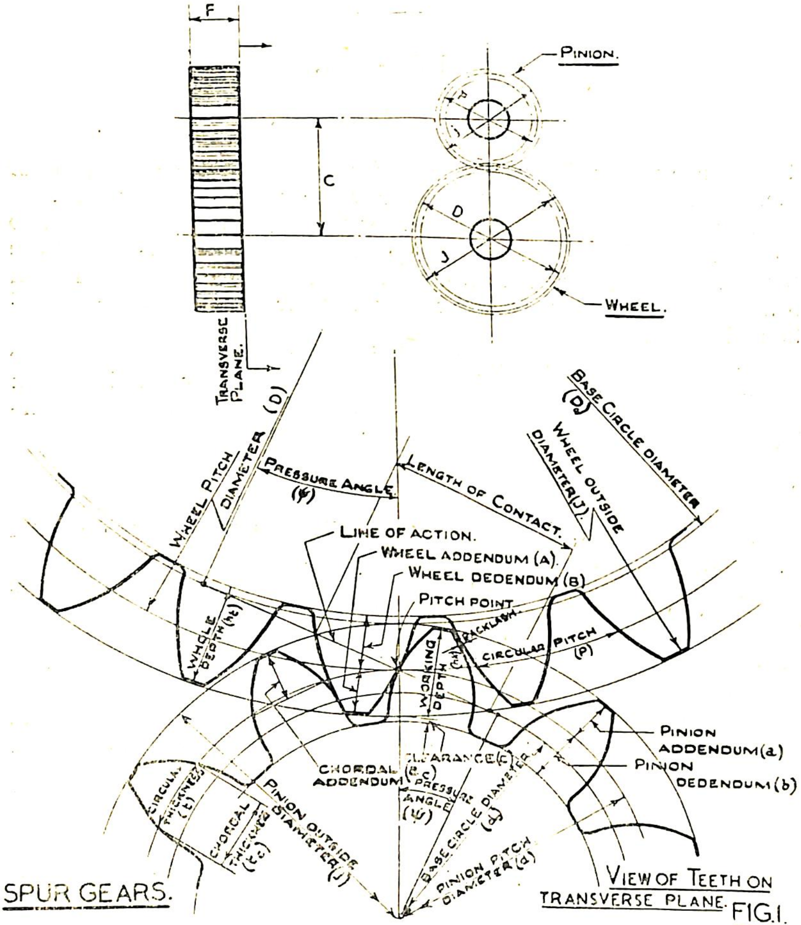
Internal Spur Gears. Fig. 2.

In this case the teeth are cut on the internal diameter of an annulus or ring, and they mesh with a spur pinion as shown in Fig. 1.

The teeth are parallel to the axis, and, as in the case of spur gears, are identical in profile from one side of the face to the other.

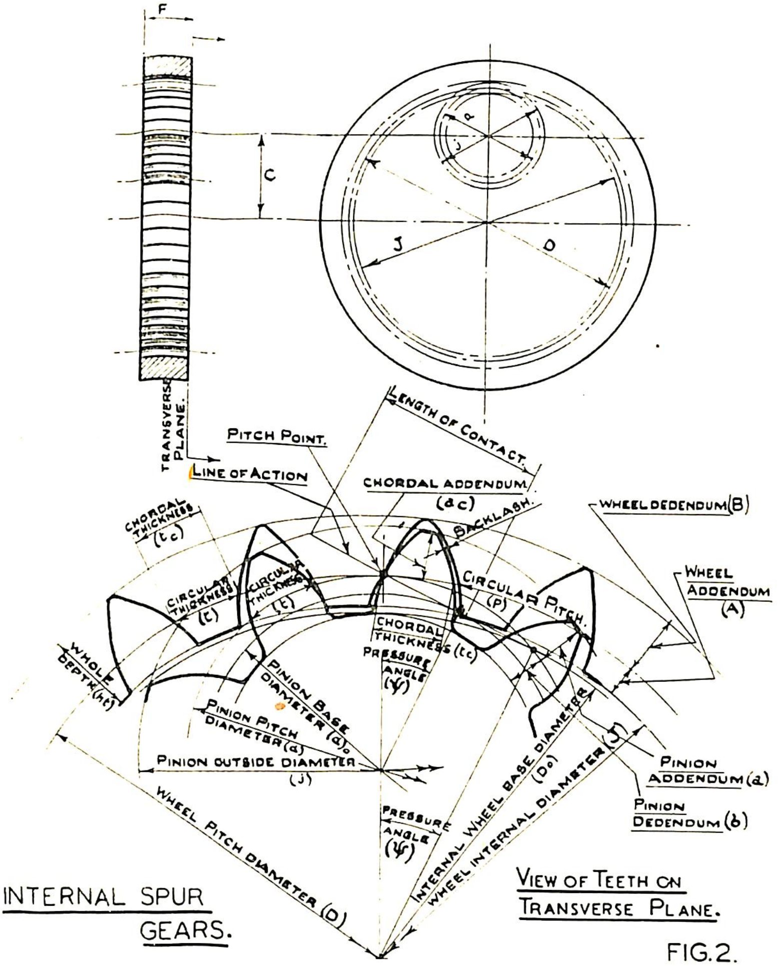
The gears are used to connect two parallel shafts usually opposed, and when engaged, the shafts run in the same direction.

This type of gear is used for epicyclic transmissions, reduction gears for machine tools, and is also used for clutches and couplings.



SPUR GEARS.

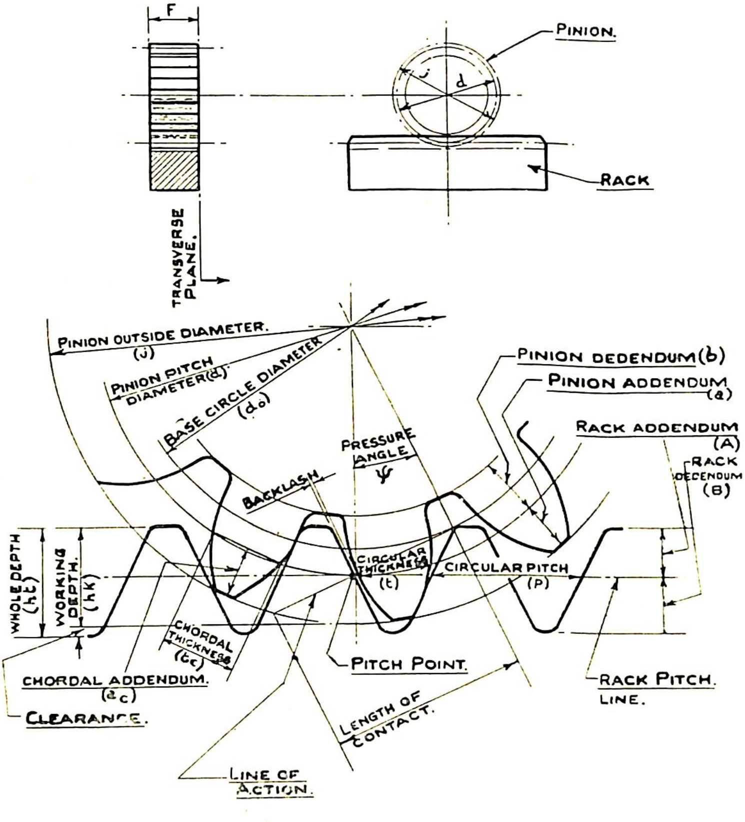
VIEW OF TEETH ON TRANSVERSE PLANE. FIG. 1.



INTERNAL SPUR GEARS.

VIEW OF TEETH ON TRANSVERSE PLANE.

FIG. 2.



VIEW OF TEETH ON TRANSVERSE PLANE.

SPUR RACK AND PINION.

FIG. 3.

In the latter case, however, the external spur pinion has the same number of teeth as the internal gear wheel, and therefore rotates as one body, at the same speed.

Spur Rack. Fig. 3.

The pitch line of a spur rack moves in a straight line path, and the rack may thus be regarded as a spur gear of infinite radius or zero curvature.

A spur pinion, Fig. 1, is most commonly used to transmit longitudinal motion to the rack, and a most common application can be found on lathes, when in this case the rack is used to transmit motion to the saddle.

It should also be stated that the rack tooth is of fundamental importance in the study of involute gear teeth.

Helical Gears. Fig. 4.

These are used in the same way as spur gears, but differ from them in that the teeth, cut on the periphery of the gear blank, are of helical or screw form.

The pinion cannot be moved longitudinally along its axis without imparting angular motion to its mating wheel and for this reason, helical gears are used in automobile gearboxes as constant mesh gears on syncro-mesh applications.

The transverse section of a helical gear is identical with a spur gear in so far as tooth profile is concerned, and is involute on this section.

Mating gears must, however, be made with the same helix angle but opposite hand, while tooth loading produces axial thrust.

Double Helical Gears. Fig. 5.

As the name implies these gears are the equivalent of two helical gears secured together, but in actual practice they are manufactured in one piece.

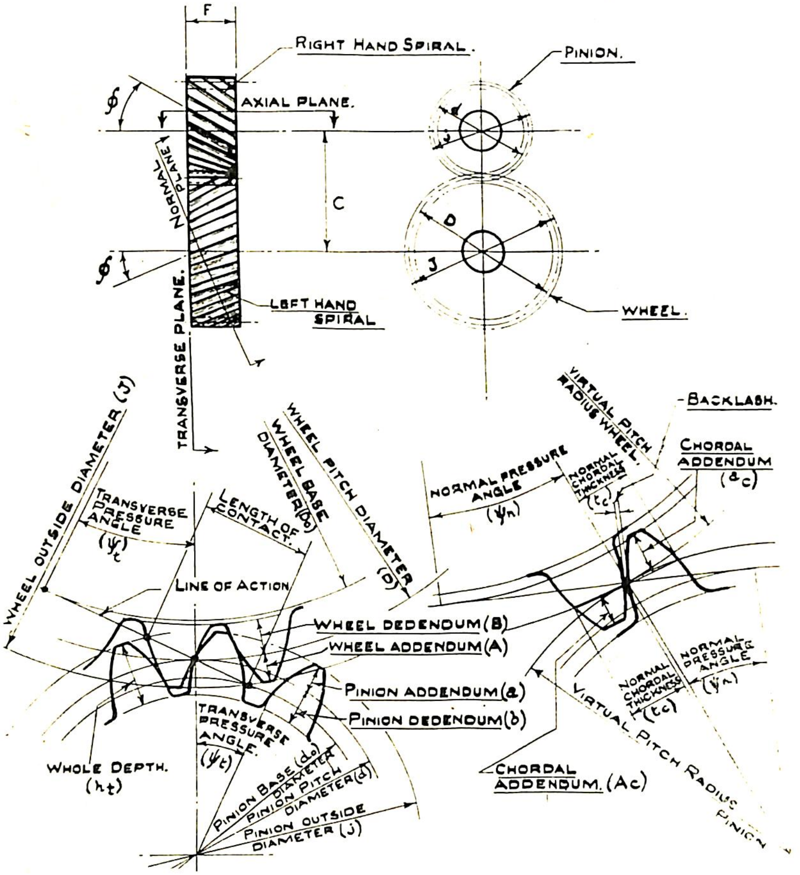
The teeth may be continuous or separated by a gap, although the two helix forming the double helical are opposite hand, meeting at a common apex.

They are produced by a variety of processes, and are distinguished by the name of the process by which the teeth are cut.

It is the usual practice when cutting this type of gear to know the direction of the teeth in relation to the driving end especially in the case of shaft gears.

Spiral Gears. Fig. 6.

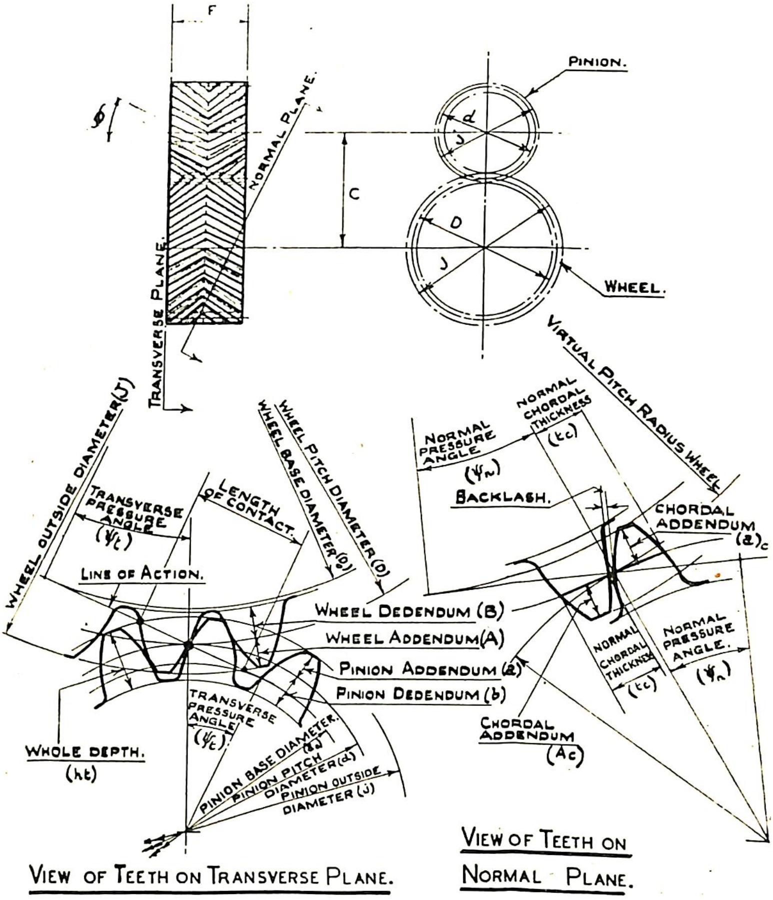
These gears are identical in every way to helical gears, Fig. 4, the only reason for them being called spiral gears is that the two


F
Right Hand Spiral.
Pinion.
Axial Plane.
ϕ
Normal Plane
C
d
j
D
J
Left Hand Spiral
Wheel.
Transverse Plane.
Wheel Outside Diameter (J)
Transverse Pressure Angle (ψ_t)
Length of Contact.
Wheel Base Diameter (D_o)
Wheel Pitch Diameter (D)
Line of Action
Wheel Dedendum (B)
Wheel Addendum (A)
Pinion Addendum (a)
Pinion Dedendum (b)
Whole Depth. (h_t)
Transverse Pressure Angle. (ψ_t)
Pinion Base (d_o) Diameter
Pinion Pitch Diameter (d)
Pinion Outside Diameter (j)
Virtual Pitch Radius Wheel.
Backlash.
Chordal Addendum (a_c)
Normal Pressure Angle (ψ_n)
Normal Chordal Thickness (t_c)
Normal Pressure Angle (ψ_n)
Normal Chordal Thickness (t_c)
Virtual Pitch Radius Pinion
Chordal Addendum. (A_c)

View of Teeth on Transverse Plane.

View of Teeth on Normal Plane.

FIG. 4.


F
φ
NORMAL PLANE.
TRANSVERSE PLANE.
C
PINION.
d
J
D
J
WHEEL.
VIRTUAL PITCH RADIUS WHEEL
WHEEL OUTSIDE DIAMETER (J)
TRANSVERSE PRESSURE ANGLE (ψ_t)
LENGTH OF CONTACT.
WHEEL BASE DIAMETER (D_b)
WHEEL PITCH DIAMETER (D)
LINE OF ACTION.
WHEEL DEDENDUM (B)
WHEEL ADDENDUM (A)
PINION ADDENDUM (a)
PINION DEDENDUM (b)
TRANSVERSE PRESSURE ANGLE (ψ_t)
WHOLE DEPTH. (h_t)
PINION BASE DIAMETER (d_b)
PINION PITCH DIAMETER (d)
PINION OUTSIDE DIAMETER (j)
NORMAL PRESSURE ANGLE (ψ_n)
NORMAL CHORDAL THICKNESS (t_c)
BACKLASH.
CHORDAL ADDENDUM (a)$_c$
NORMAL CHORDAL THICKNESS (t_c)
NORMAL PRESSURE ANGLE. (ψ_n)
CHORDAL ADDENDUM (A_c)

VIEW OF TEETH ON TRANSVERSE PLANE.

VIEW OF TEETH ON NORMAL PLANE.

FIG. 5.

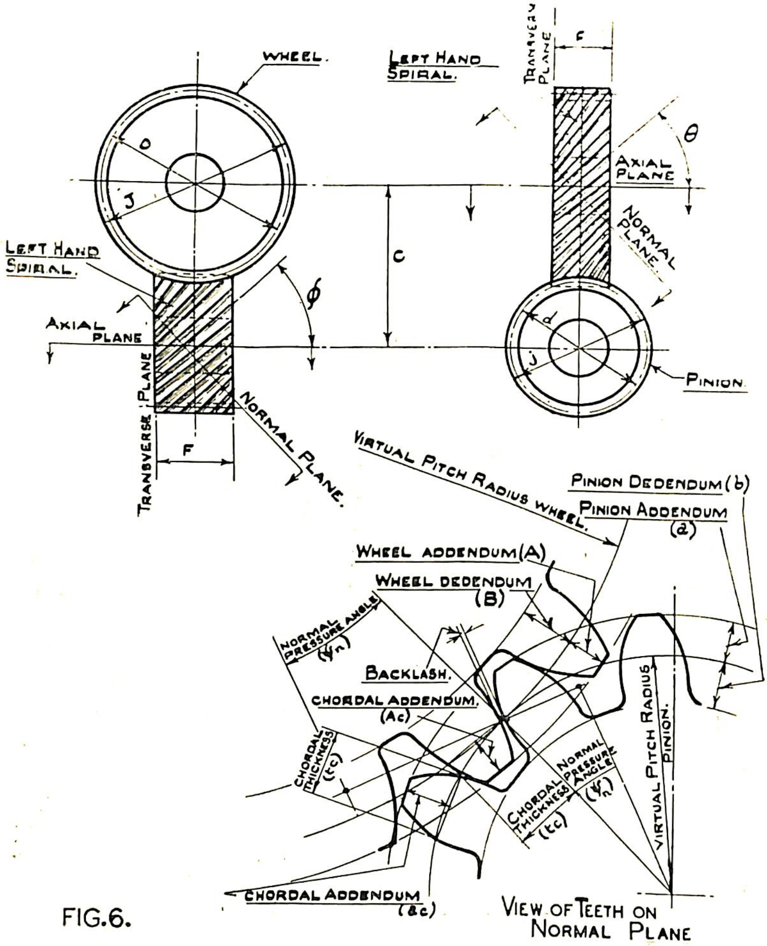

WHEEL.
LEFT HAND SPIRAL.
TRANSVERSE PLANE
F
θ
AXIAL PLANE
NORMAL PLANE.
D
J
C
φ
LEFT HAND SPIRAL.
AXIAL PLANE
TRANSVERSE PLANE
F
NORMAL PLANE.
d
j
PINION.
VIRTUAL PITCH RADIUS WHEEL.
PINION DEDENDUM (b)
PINION ADDENDUM (a)
WHEEL ADDENDUM (A)
WHEEL DEDENDUM (B)
NORMAL PRESSURE ANGLE (ψ_n)
BACKLASH.
CHORDAL ADDENDUM. (Ac)
CHORDAL THICKNESS (Tc)
CHORDAL THICKNESS (tc)
NORMAL PRESSURE ANGLE (ψ_n)
VIRTUAL PITCH RADIUS PINION.
CHORDAL ADDENDUM (ac)
VIEW OF TEETH ON NORMAL PLANE

FIG. 6.

gears are used to connect two shafts not parallel to each other, and this difference should be particularly noted.

Spiral gears, therefore, used in this way give only theoretical point contact instead of line contact which has been the case for the gears previously mentioned, and they also introduce a longitudinal sliding motion between the teeth.

Bevel Gears. Fig. 7.

When it is desired to connect two shafts the axis of which intersect, bevel gears are used.

The most common type is that in which the teeth are radial to the point of intersection of the shaft axes or apex, and such gears are termed straight bevel gears.

The teeth are similar to spur gears in that they make line contact across the face of the teeth. The teeth are smaller at the front end than the rear end, being smaller in proportion to the rear end in the ratio of (1) distance from apex to the front end; (2) the distance from the apex to the rear face.

The shaft angle most commonly encountered is 90° and bevel gears having this angle when giving equal speeds are termed mitre gears.

The shaft angle may, however, be any value between zero and 180°, but recourse should be made to the limitations of the machine used for cutting the gears before anything definite is fixed.

Spiral Bevel Gears. Fig. 8.

This type of gear fulfils the same purpose as straight bevel gears, but represents an extension of the principle of tooth formation to the general case in which the teeth may have a form of spiral or developed curve in the longitudinal direction.

In other words, spiral bevel gears bear the same relation to straight bevel gears as helical gears bear to spur gears.

There is no longitudinal sliding, but the end thrust may act either away from or towards the apex according to the direction of rotation, hand of spiral, and gear proportions.

In the actual manufacture of this type of gear the design depends on the type of machine used for cutting the teeth, and furthermore, the actual settings used on that machine are important as a means of reproduction should further quantities be required.

Worm Gears. Fig. 9.

Worm gears are used in the same manner as spiral gears, *i.e.*, to connect skew shafts, but not necessarily at right angles.

In fact, the worm exactly resembles one of a pair of spiral gears, but the wheel is throated, and has concave teeth.

While resembling spiral gears in purpose and general dimensions, worm gears differ from them in that they give line contact instead of point contact, and will thus carry greater loads. Another difference is that while the centre distance is important in relation to the helix angle in the case of spiral gears, this problem does not arise when making calculations for worm gears.

The sliding velocity is high in comparison with that found in other types of gear, and special attention is necessary to the choice of materials and lubricant.

CHAPTER II.

FORM OF TOOTH.

The function of a pair of gears is to transmit motion from one shaft to another with uniformity of velocity ratio.

Either the cycloidal, or the involute form of tooth, will satisfy this condition, but the cycloidal has the disadvantage in that a slight variation of the theoretical centre distance destroys the uniformity of angular velocity.

It is very difficult to ensure that two mating gears run on their exact theoretical centres under all circumstances, and it is this fact more than any other which has put the involute tooth form into almost universal use.

Variations within fairly wide limits of the theoretical centre distance of two involute tooth form gears does not impair the uniformity of velocity ratio.

Another factor in favour of the involute is that it can be measured by means of two very simple pieces of mechanism, namely, a straight edge and disc.

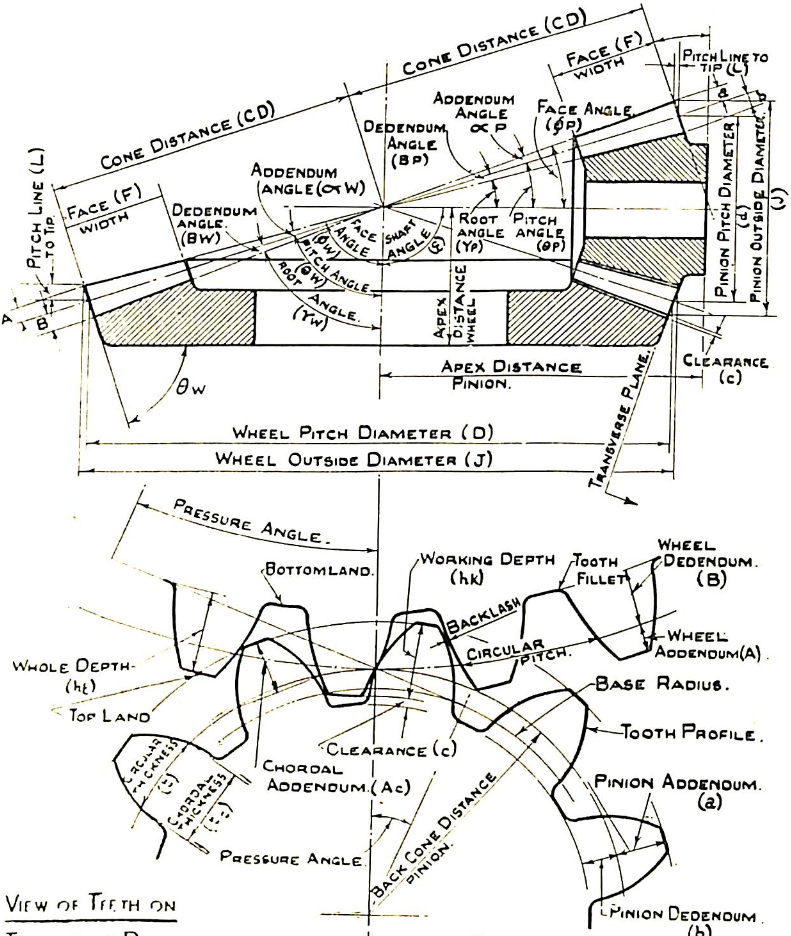
A third factor is that an involute tooth form can be produced by a straight sided rack cutter, and, in fact, every type of involute gear is originally produced by means of this basic principle.

It can, therefore, be stated that all the types of gears described in the previous chapter are produced with an involute tooth form with the exception of the bevel gears, Fig. 7, and spiral bevel gears, Fig. 8.

In the case of bevel gears, the teeth are generated by a straight sided cutter, representing the side of a tooth of any imaginary crown gear, and the form of tooth produced is termed an octoid.

Now a crown gear is in effect a bevel gear of 90° pitch angle, and the teeth of a true involute crown gear should have tooth profiles which are very slightly curved.

The difference between the tooth shapes on the gear being cut by an imaginary crown gear having a straight sided cutter, and



VIEW OF TEETH ON TRANSVERSE PLANE.

FIG. 7.

one with the tooth profile which is slightly curved, is so small however, that it can be ignored.

Both forms give theoretically, correct action, and for practical purposes it can safely be assumed that the form of tooth produced on the transverse plane of both straight and spiral bevel gears is of involute form.

Spur gears, spur racks, helical, spiral gears and worms, are all of involute form on their transverse planes or in other words, on the plane of rotation.

This point should be kept in mind when making any investigations, calculations, or layouts appertaining to the form of tooth, especially in the case of helical, spiral gears or worms.

For these types of gears, which in any case are very closely related to each other, the form of tooth on the normal and axial planes is such that the form is entirely dependent on the involute form on the transverse plane, and the spiral or helix angle.

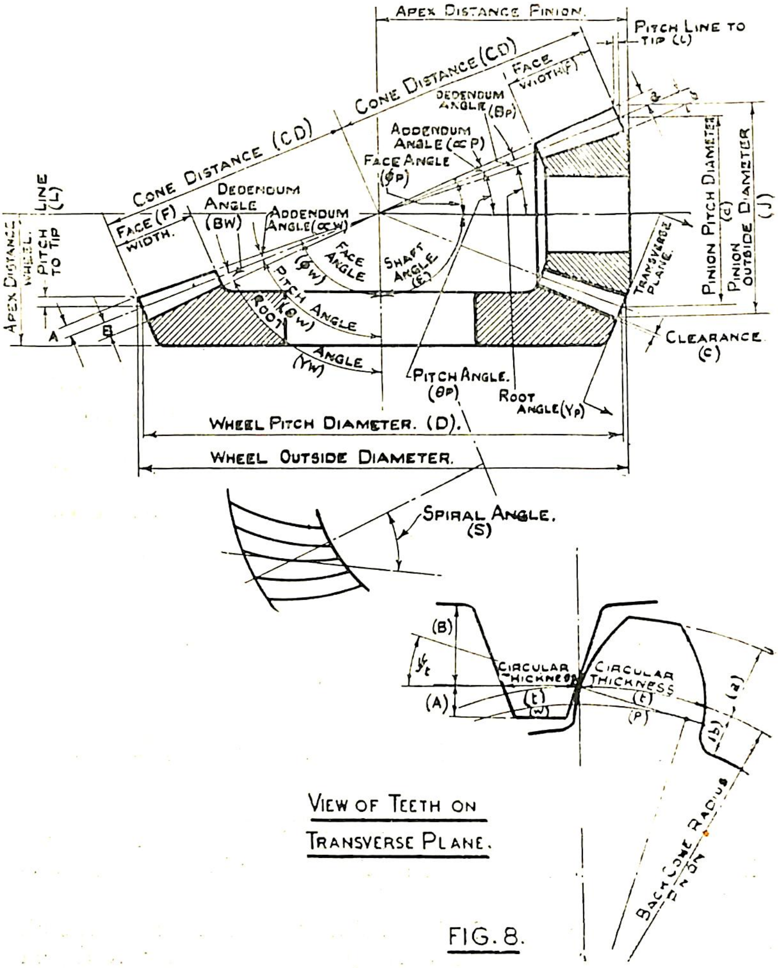


FIG. 8.

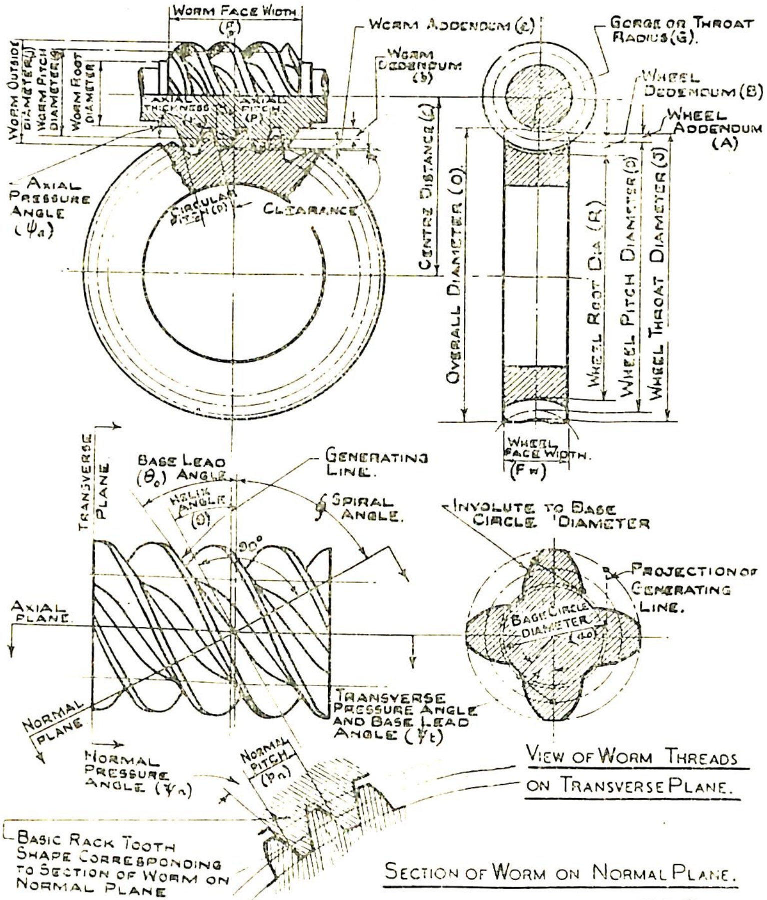

Worm Face Width
(F_p)
Worm Outside Diameter (J)
Worm Pitch Diameter (d)
Worm Root Diameter
Axial Thickness (t)
Axial Pitch (P)
Worm Addendum (a)
Worm Dedendum (b)
Gorge or Throat Radius (G).
Wheel Dedendum (B)
Wheel Addendum (A)
Axial Pressure Angle (ψ_a)
Circular Pitch (p)
Clearance
Centre Distance (C)
Overall Diameter (O).
Wheel Root Dia (R)
Wheel Pitch Diameter (D)
Wheel Throat Diameter (J)
Wheel Face Width.
(Fw)
Transverse Plane
Base Lead (θ_0) Angle
Helix Angle (θ)
Generating Line.
Spiral Angle.
90°
Axial Plane
Normal Plane
Transverse Pressure Angle and Base Lead Angle (ψ_t)
Involute to Base Circle Diameter
Projection of Generating Line.
Base Circle Diameter (d_0)
Normal Pressure Angle (ψ_n)
Normal Pitch (p_n)
Basic Rack Tooth Shape Corresponding to Section of Worm on Normal Plane
View of Worm Threads on Transverse Plane.
Section of Worm on Normal Plane.

FIG. 9.

LAYOUT OF INVOLUTE CURVES.

The involute tooth form has been constructed on the transverse plane of the spur gear pair shown in Fig. 1, and it will be particularly noted that the origin of the wheel and pinion tooth shapes is based on the base circle diameters D_o and d_o.

The base circle diameter should be considered as a disc, around which is a line or tape, the two ends of the tape commencing and finishing at point A as shown in Fig. 10.

By securing one end of the tape at A and unwinding the other end, the path of the involute curve is described by the end of this tape, through points B, C, D and E, when the tape is tangential to the base disc at the corresponding points B_1, C_1, D_1, and E_1.

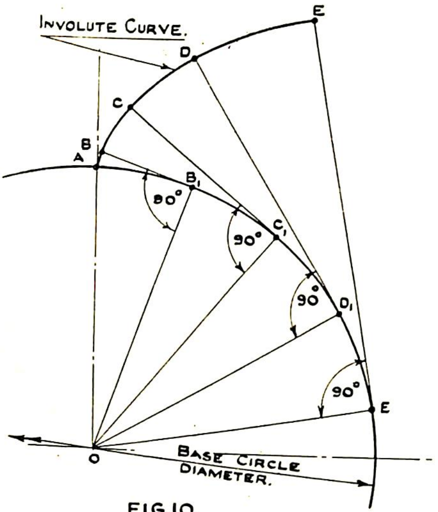


FIG. 10.

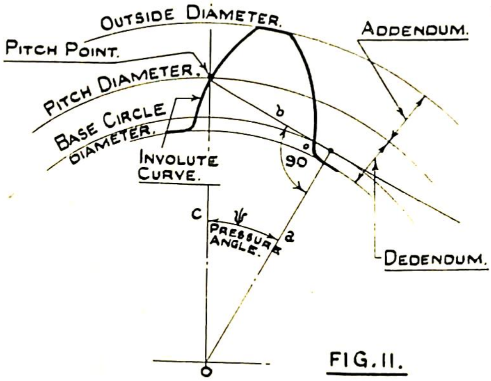


FIG. 11.

Before, however, the involute form can be applied to a gear tooth, some consideration should be given to the question of the pitch diameter, and pressure angle.

In other words, the pitch diameter and pressure angle together with the addendum and dedendum actually define what part of the involute curve is being utilised on a gear tooth, and before any tooth layout can be made it is first of all necessary to have this information.

The pitch diameter is always greater than the base diameter, and where the involute bisects the pitch diameter circle, it is called the pitch point, while from this point the pressure angle is obtained in conjunction with the base circle, and the centre of the gear.

In Fig. 11 is shown the construction of a spur gear tooth in relation to the pressure angle and pitch diameter, and it should be noted that we have a right angle triangle formed by the three sides a, b and c when $a = \frac{1}{2}$-base circle diameter.
$c = \frac{1}{2}$-pitch diameter.
$\psi =$ pressure angle of gear.

Therefore given the pitch diameter D and pressure angle ψ of a gear, the base circle diameter D_o is equal to D Cos ψ when $D = 2_c$ and $D_o = 2_a$. Alternatively given D and D_o the pressure angle ψ can be obtained as follows :—

$$\cos \psi = \frac{D_o}{D}$$

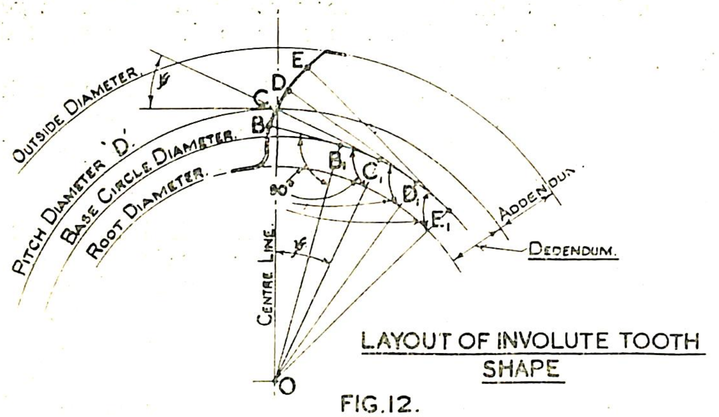
There are several methods of setting out the involute tooth form on the drawing board, and the method selected depends on the accuracy required for the purpose for which it is proposed to use the tooth shape.

For example, if an accurate projection drawing is required, it is recommended that the tooth shape be generated, especially if a gear having a small number of teeth is under review, as the shape changes very rapidly on gears of this nature.

For a gear having a large number of teeth, then the shape can be constructed without being generated as the tooth shape does not change, and, in fact, it will be found that the shape is practically equivalent to the arc of a circle.

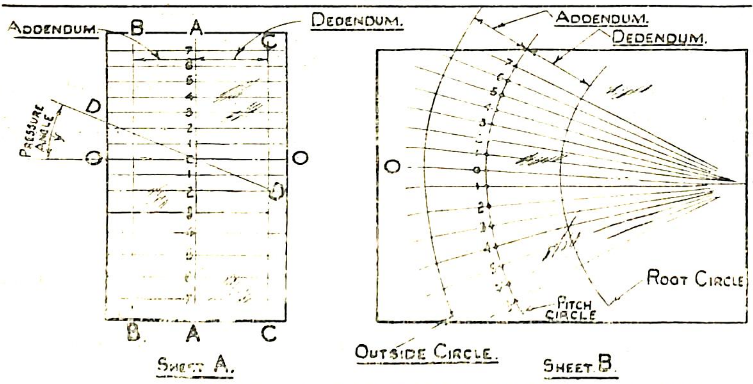
For setting out tooth shapes, when accuracy is not the first essential, the following procedure can be followed, this method being suitable for helical, spiral, worm, and bevel gears on their transverse planes, as well as spur gears.

Referring to Fig. 12, the first step is to draw the pitch diameter D, and the outside and root circles, the outside circle being larger in diameter than the pitch diameter, by an amount equal to twice the addendum, while the root circle is smaller in diameter than the pitch circle by an amount equal to twice the dedendum of the gear.



LAYOUT OF INVOLUTE TOOTH SHAPE

FIG. 12.



GENERATION OF TOOTH SHAPE. FIG. 13.

The pressure angle ψ is then set out from the centre line of the gear and from the gear centre "O" while another line from the pitch point "C" at right angles to line OC_1 is drawn to form a triangle OCC_1, after which the base circle is drawn equal to a radius of OC_1.

The compasses are then set to distance C_1C and an arc drawn from C to D.

The compass point is then placed on D_1, the compasses extended to reach point D and an arc drawn to reach point E.

This process is repeated until the involute curve reaches the outside circle and a similar procedure is adopted for the curve below the pitch diameter to the base circle except that in this case the radii of the compasses is shortened instead of lengthened.

If the root circle diameter is less than the base circle diameter then the involute curve must not be taken below the base circle, due to the fact that the curve terminates at this point.

To determine the generated tooth shape below the base circle it is therefore necessary to generate the tooth shape as described below.

If, however, the distance is equal to the radius used to join up the flank of the teeth to the root radius, then it will, of course, not be necessary to generate this radius, which is usually made equal to the clearance.

It must be understood that although only four points have been used in constructing the involute curve in the figure, more points could be used and, in fact, greater accuracy will be obtained, but all this, of course, depends upon the size of the tooth being set out.

The other side of the tooth is drawn in the same way after setting off the tooth thickness on the pitch line, the arcs used to construct the curve being exactly the same as were used for the first side.

A more accurate method for setting out the tooth shape is to "generate" the curve on the drawing board, and this can be carried out in several ways.

They are all similar, however, and two ways are described below, but it should be specially noted that it is in effect only reproducing the generating action of the gear cutting machine, on the drawing board.

For one of the methods used for generating the tooth shape two sheets of drawing paper are required, sheet A and B, Fig. 13.

On sheet A a vertical line AA is drawn, and at right angles to this line a centre line OO is drawn. Two further lines BB and CC are then drawn, parallel to line AA, the distances from line AA of lines BB and CC being equal to the addendum and dedendum of the gear respectively.

With line OO as centre a series of lines parallel to OO are then drawn, above and below line OO, and numbered as shown, the spacing of these lines being made equidistant by the use of a pair of dividers.

The distance between the lines depends on the size of the drawing required but for general cases it will be found that a distance of 0·300 to 0·500 will be satisfactory.

To complete the drawing another line DD corresponding to the rack form is then drawn, the angle ψ to line OO being made equal to the pressure angle.

For the drawing on sheet B, transparent or tracing paper must be used, and after first drawing the centre line OO the circles corresponding to the pitch circle, outside circle, and root circle diameters are drawn.

The pitch circle is then divided up into the same distances as for line AA on sheet A, and it is advisable to keep the dividers set for marking off both sheets.

Having set out the required number of points on the pitch circle sheet B above and below centre line OO, radial lines are then drawn from each point to the centre O and numbered as shown.

To "generate" the tooth shape sheet B is placed over sheet A with the horizontal lines OO covering each other and line AA, sheet A, tangential to the pitch circle diameter sheet B, after which the angular line DD sheet A is traced on to sheet B.

Still keeping line AA tangential to the pitch circle diameter, line number 1 sheet A is covered by line number 1 sheet B, and line DD sheet A traced through on to sheet B in this new position.

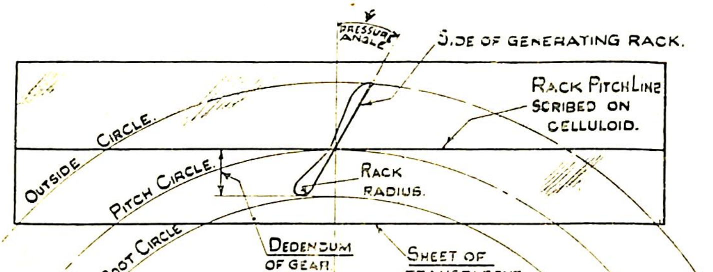


FIG. 14.

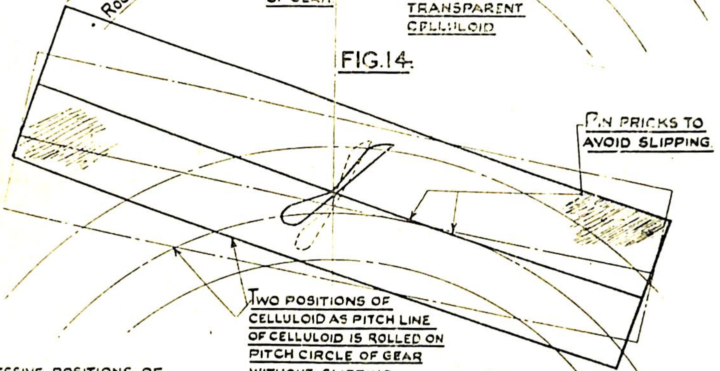

PIN PRICKS TO AVOID SLIPPING.
TWO POSITIONS OF CELLULOID AS PITCH LINE OF CELLULOID IS ROLLED ON PITCH CIRCLE OF GEAR WITHOUT SLIPPING.

FIG. 15.

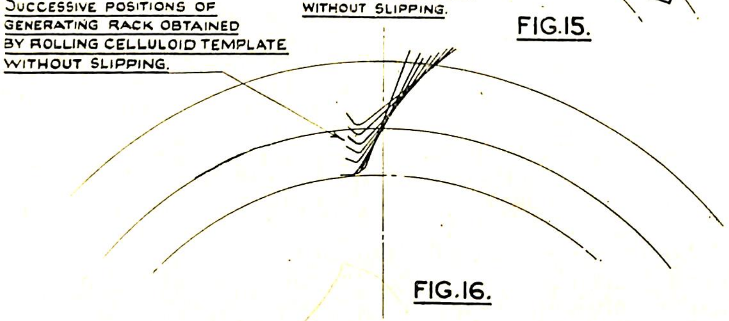

SUCCESSIVE POSITIONS OF GENERATING RACK OBTAINED BY ROLLING CELLULOID TEMPLATE WITHOUT SLIPPING.

FIG. 16.

This procedure is followed for each numbered line on the sheet both above and below the centre line OO until the whole of the involute curve has been produced.

Another similar method of generating the tooth shape is to first produce the rack form on a piece of thin transparent celluloid as shown in Fig. 14.

The advantage of this method will be found when it is desired to generate a wheel tooth from a pinion tooth or *vice versa*, as in this case the profile of the pinion tooth will be produced on the celluloid and rolled with the wheel in the same manner, although it will be necessary to scribe the pinion pitch circle on the celluloid and not a straight line as shown.

This method is found very useful when it is desired to know the shape of a spur gear tooth generated by a circular generating cutter, the cutter shape and cutter pitch circle being first produced on the celluloid, and the spur gear tooth generated from this shape by rolling the two pitch circles together.

A pencil is used to mark the side of the rack or cutter form on the drawing paper, with the rack pitch line tangential to the pitch circle, and pin pricks made to avoid slipping.

The template is then rolled on the pitch circle without slip keeping the celluloid pitch line tangential to the pitch circle, further pin pricks being made on the pitch circles, and another line marked on the drawing.

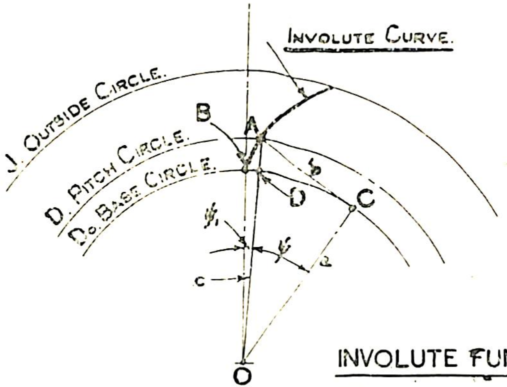
This procedure is continued until the gear tooth shape is produced on the drawing, the shape above the pitch circle being produced by turning the template clockwise and the shape below the pitch circle is produced by turning the template anti-clockwise, when the completed tooth shape will be as shown at Fig. 16 if the generation has been carried out correctly.

INVOLUTE FUNCTIONS.

In addition to being able to generate gear tooth forms on their transverse planes in the manner described, it is also possible to plot the tooth form, using co-ordinates previously calculated by means of involute trigonometry in conjunction with which involute functions are used.

To define an involute function, an involute tooth form is shown at Fig. 17, where

ψ = pressure angle of gear on transverse plane at radius D/2.
D = pitch line diameter.
D_o = base line diameter.
J = outside diameter.
ψ_1 = involute function of angle ψ.



INVOLUTE FUNCTION.

FIG. 17.

The right angle used in the construction is denoted by three sides *a*, *b* and *c*, and if it is imagined that side *b* is wound round the base circle until point A reaches the base circle at point B as previously explained, it is evident therefore that *b* will equal the circular distance BC measured on the base circle.

Now an involute function is the vectorial angle (in radians) of an involute curve from its origin, and is also a function of the pressure angle at the radius, at which the vectorial angle is taken, the pressure angle at the point of origin B being zero.

The vectorial angle at A on the involute curve is therefore ψ_1, which is the angle subtended by OB and OA, the pressure angle at A at radius D/2 being ψ.

Therefore the angle ψ_1, in radians is known as the involute function of the angle ψ and is written inv. ψ.

$$\text{Now } \frac{b}{a} = \tan\psi \text{ and } b = a\tan\psi.$$

Length *b* is then rewound on to the base circle diameter and is then equal to $a\ (\psi + \psi_1)$.

$$\begin{aligned}\text{Therefore } a\ (\psi + \psi_1) &= a \tan \psi.\\ \psi + \psi_1 &= \tan \psi.\\ \psi_1 &= \tan \psi - \psi \text{ radians.}\\ \psi_1 &= \text{inv. } \psi.\end{aligned}$$

Example.—To find the involute function of 20° we proceed as follows :—

$$\begin{aligned}\text{Let } \psi &= 20^\circ.\\ ,,\ \psi_1 &= \text{inv. } 20^\circ = \tan 20^\circ - 20^\circ \text{ radians.}\end{aligned}$$

Then $\tan 20^\circ = 0{\cdot}363968$ and 20° radians $= 0{\cdot}349064$.

$$\begin{aligned}\therefore\ \text{Inv. } 20^\circ &= 0{\cdot}363968 - 0{\cdot}349064\\ &= 0{\cdot}014904.\end{aligned}$$

CHAPTER III.

CORRECTION OF SPUR GEARS.

There are many methods employed in the correction of spur gear teeth produced by generation, all of which however are used to eliminate undercutting of the tooth flanks, and thereby improve the running qualities of the gears.

The same methods can be used in the design of single helical, and spiral gears of full depth or British Standard proportions (based on the normal plane), and produced by hobbing, rack planing or profile grinding, providing that the correction is based on the virtual and not the actual number of teeth in the gear, when

$Z_v = Z \sec^3 \theta$.
and Z_v = virtual number of teeth in gear.
Z = actual number of teeth.
θ = spiral angle.

There are four methods of tooth correction in common use as described below :—

(*a*) Shortening the addendum and dedendum of both wheel and pinion, the teeth then being known as stub teeth.

(*b* Increasing the pressure angle of the gears.

(*c*) Displacing the pinion tooth form outwards, and the wheel tooth form inwards, by a similar amount.

(*d*) Increasing centre distance, and displacing tooth profile of both pinion and wheel outwards, to suit the increased centre distance.

Stub Teeth.

The stub tooth system is a modification of the full depth involute tooth, and is used extensively for gears in automobile and machine tool gearboxes.

Its advantage lies in the fact that added strength is given to the teeth, but against this, it will be found that the length of contact is shortened, and it is therefore advisable to check this either by layout or calculation, to ensure continuity of contact.

There are also various standards in use for stub teeth, details of the basic rack for each standard being given below.

Standard	*Addendum*	*Dedendum*	*Full Depth*	*Radii in Roots*	*Tooth Thickness*
Fellows	—	—	—	—	—
American	$\frac{0{\cdot}800}{DP}$	$\frac{1{\cdot}000}{DP}$	$\frac{1{\cdot}800}{DP}$	$\frac{0{\cdot}30}{DP}$ Min	$\frac{\pi}{2DP}$
Nuttall's	$\frac{0{\cdot}7854}{DP}$	$\frac{0{\cdot}9424}{DP}$	$\frac{1{\cdot}7278}{DP}$	$\frac{0{\cdot}25}{DP}$	$\frac{\pi}{2DP}$
Adamson's	$\frac{0{\cdot}7854}{DP}$	$\frac{0{\cdot}9425}{DP}$	$\frac{1{\cdot}7279}{DP}$	$\frac{0{\cdot}25}{DP}$	$\frac{\pi}{2DP}$

For practical purposes the Nuttall and Adamson standards are the same although the Nuttall system is generally based on using the circular pitch proportions thus :—

addendum = 0·250 × circular pitch.
dedendum = 0·300 × circular pitch.

No set rule however can be fixed for the Fellows system, which is the most generally used in this country, as it is based on the use of two diametral pitches.

For example when a pitch of $\frac{5}{7}$ diametral pitch is quoted, it means that the diametral pitch of the gear is 5 diametral pitch for the purpose of calculating the pitch circle diameter and circular tooth thickness, but the addendum, dedendum, and subsequently the outside and root diameters are based on the standard proportions of 7 diametral pitch.

Generating cutters designed for the full depth involute system cannot be used, and special cutters are therefore required, based on the stub tooth standard.

The basic pressure angle used for all stub tooth standards is 20°, while the diametral pitches included in the Fellows system are as follows :

$\frac{4}{5}$, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{7}{9}$, $\frac{8}{10}$, $\frac{9}{11}$, $\frac{10}{12}$, $\frac{12}{14}$, $\frac{14}{18}$, $\frac{16}{21}$ and $\frac{18}{21}$, etc.

Example.—Calculate the tooth proportions of stub tooth gear $\frac{8}{10}$ DP, 32 teeth, 20° pressure angle.

$$D = \frac{32}{8} = 4{\cdot}000$$

$$A = \frac{1}{10} = 0{\cdot}100$$

$$B = \frac{1{\cdot}250}{10} = 0{\cdot}125$$

$$J = 4{\cdot}000 + 2 \times 0{\cdot}100 = 4{\cdot}200$$

$$t = \frac{\pi}{2 \times 8} = 0{\cdot}19635$$

when D = pitch circle diameter.
A = addendum.
B = dedendum.
J = outside diameter.
t = circular tooth thickness.

other particulars required for cutting the gear being calculated in the same manner as for a standard spur gear.

It is generally accepted that all spur gears having 17 teeth and over, 20° pressure angle, could be made with standard full depth proportions; therefore if the teeth are made to the stub tooth standard it will be possible to design a gear of less than 17 teeth without undercut.

To determine if undercut is present it is necessary to make a small calculation, and referring to Fig. 18 :—

let D = pitch circle diameter.
B_w = working dedendum of gear.
ψ = pressure angle.
x = amount of involute profile below pitch circle.
k = correction required to eliminate undercut.

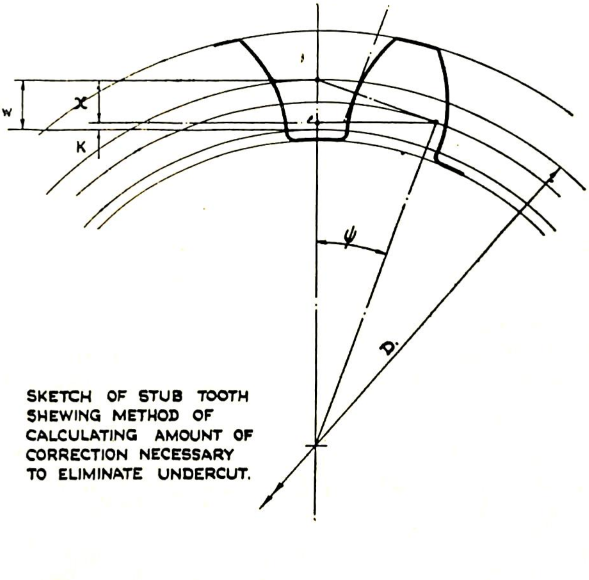


FIG. 18. SKETCH OF STUB TOOTH SHEWING METHOD OF CALCULATING AMOUNT OF CORRECTION NECESSARY TO ELIMINATE UNDERCUT.

$$\text{then } x = \frac{D}{2} \sin^2 \psi.$$

thus if x is less than B_w the difference is equal to k, the amount of correction required.

Taking the previous example the pitch circle diameter is 4·000 and B_w 0·100, then

$$x = \frac{4{\cdot}000}{2} \sin^2 20^\circ$$

$$= 0{\cdot}233$$

but in this case 0·233 is greater than 0·100, therefore no correction is necessary. With the Fellows stub tooth system it is not possible to fix the minimum number of teeth that can be cut without undercut, but taking the largest pitch in the system, namely 4/5 DP, the minimum number of teeth can be calculated.

$$B_w = 0{\cdot}200.$$

To avoid correction x must also equal 0·200,

$$\text{then } \frac{D}{2} = \frac{x}{\text{Sin}^2\psi} = \frac{0{\cdot}200}{\text{Sin}^2\ 20^\circ}$$

$$\text{or } D = \frac{0{\cdot}200 \times 2}{\text{Sin}^2\ 20^\circ}$$

$$= 3{\cdot}42$$

and minimum number of teeth

$$= D \times DP$$

$$= 3{\cdot}42 \times 4 = 13{\cdot}68$$

$$= 14 \text{ teeth.}$$

The formula given is based on the assumption that the gears are meshing with a rack, and therefore undercut must be eliminated altogether.

When the gear is meshing with another gear, however, it is not absolute necessary to do this, as the whole working depth is not being utilised, due to the curvature of the mating wheel, but this will be explained later.

Increasing Pressure Angle.

The second method used to eliminate undercutting of the tooth flanks, is to increase the pressure angle of both gears.

Its advantages are that a stronger tooth is obtained, but there are certain disadvantages, namely, a smaller engagement factor, and care has to be taken to see that the points of the teeth do not become too pointed.

The theoretical minimum number of teeth which can be cut for a given pressure angle, using standard proportions, is obtained from the formula :—

$$Z = \frac{2}{\text{Sin}^2 \psi}$$

when Z = minimum number of teeth.
ψ = pressure angle of cutter.

and it will be found that

for $14\frac{1}{2}°$	pressure	angle	this	minimum	is	32
,, 20°	,,	,,	,,	,,	,,	17
,, 25°	,,	,,	,,	,,	,,	11

In all cases the engagement factor or contact ratio should be greater than 1·000 (a figure of 1·3 to 1·5 is desirable) and this can be calculated or obtained from a layout of the gear pair under consideration.

If calculated, the contact ratio is obtained from the formula :—

$$\frac{\sqrt{J^2 - D_o{}^2} \;+\; \sqrt{j^2 - d_o{}^2} \;-\; 2C \,\text{Sin}\, \psi}{2 \times \pi/DP \times \cos \psi}$$

J = outside diameter of wheel.
j = outside diameter of pinion.
D_o = base diameter of wheel.
d_o = base diameter of pinion.
ψ = pressure angle.
DP = diametral pitch.
C = centre distance.

To obtain the contact ratio from a layout, reference should be made to Fig. 19 which shows a gear pair of standard proportions, then contact ratio is equal to :—

$$\frac{\text{Length of contact}}{\text{Base pitch}}$$

when base pitch $= \dfrac{\pi}{DP} \cos \psi$

The number of pitches in mesh at one time is expressed by the contact ratio, therefore should the contact ratio be equal to 1·000, one tooth is just leaving engagement as the next tooth is just entering.

The gear pair shown in the figures has 20 teeth in the pinion, and 24 teeth in the wheel, while the pressure angle is 20°.

The tooth proportions are therefore made standard in accordance with the rules previously given, the "length of contact" denoted by the letters EF, and the length of the line of action by the letters GH, then contact ratio is equal to :—

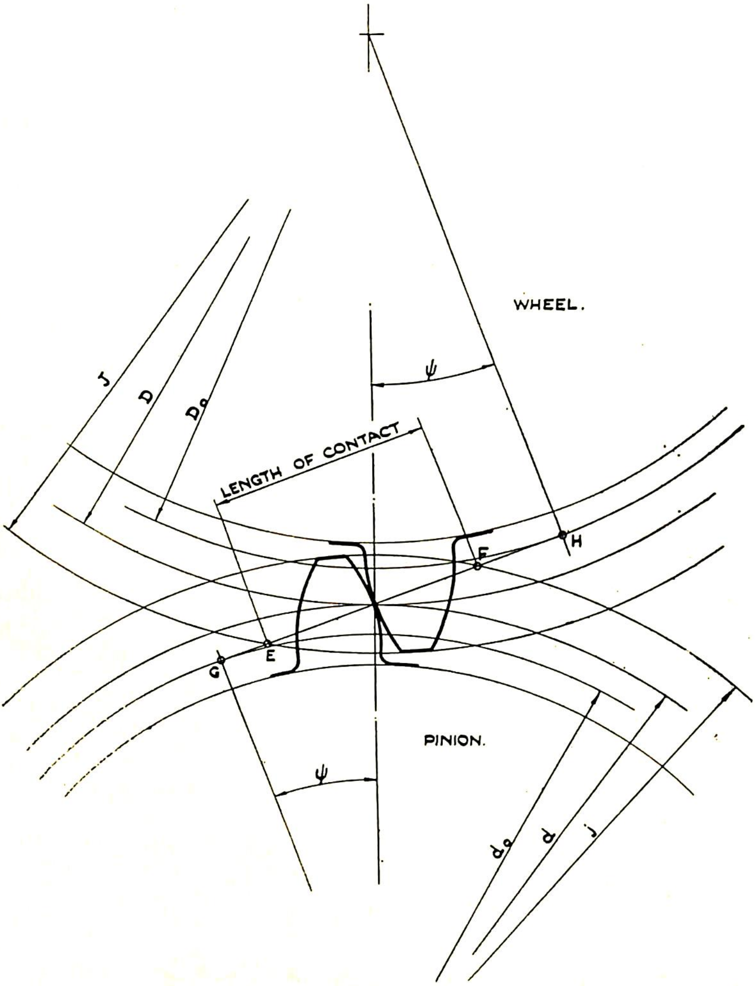


Fig. 19—Layout to obtain length of contact for a pair of spur gears.

$$\frac{EF}{\pi/DP\cos\psi} \quad \text{or} \quad \frac{EF}{\text{Base pitch}}$$

which in this layout is equal to 1·56.

When increasing the pressure angle, a check should also be made on the point width of the teeth, especially if dealing with a small number of teeth.

This can also be obtained either by layout or calculation but greater accuracy is obtained by calculation.

Then referring to Fig. 20, to obtain the circular width of the tooth on the outside diameter we proceed as follows when

J = outside diameter of gear on which thickness is required.
D = pitch diameter of gear.
D_0 = base diameter of gear.
ψ = pressure angle on diameter D.
θ = pressure angle on diameter J.

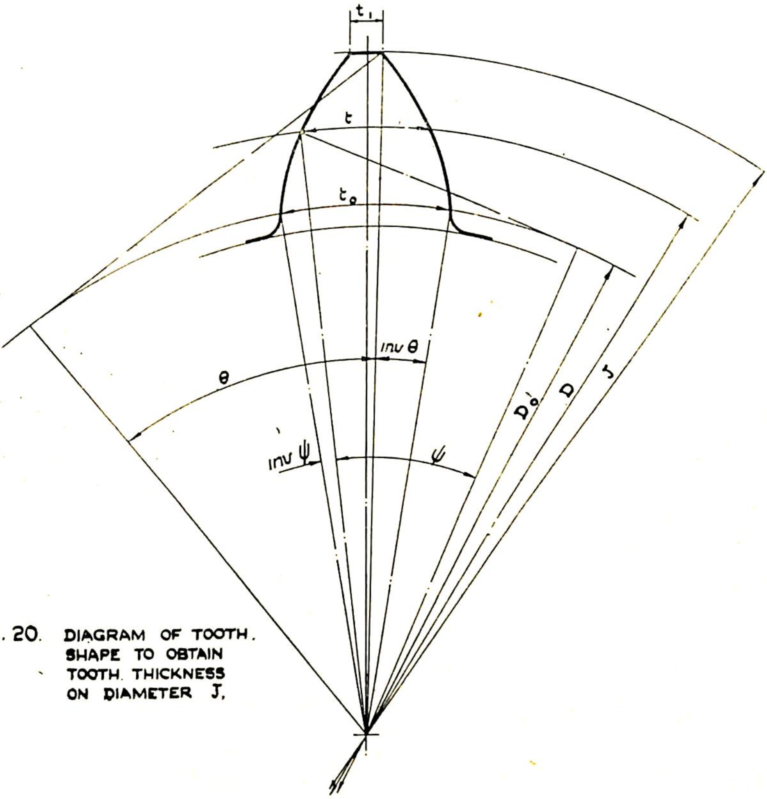


FIG. 20. DIAGRAM OF TOOTH SHAPE TO OBTAIN TOOTH THICKNESS ON DIAMETER J.

t = circular tooth thickness on diameter D.
t_1 = circular tooth thickness on diameter J.
t_o = circular tooth thickness on base diameter D_o.
inv. ψ = involute function of angle ψ } from tables.
inv. θ = involute function of angle θ } from tables.

Then $t_1 = J\left[\left(\frac{t}{D} + \text{inv. }\psi - \text{inv. }\theta\right)\right]$

$$\text{when } \cos\theta = \frac{D\cos\psi}{J} \text{ or } \frac{D_o}{J}$$

should the tooth thickness on the base circle be required we have :

$$t_o = D_o\left[\frac{t}{D} + \text{inv. }\psi\right]$$

and given t_o the tooth thickness on diameter, J can then be calculated from :—

$$t_1 = J\left[\frac{t_o}{D_o} - \text{inv. }\theta\right] \text{ or } t = D\left[\frac{t_o}{D_o} - \text{inv. }\psi\right]$$

The above formula can be used in calculating the circular tooth thickness at any diameter desired, using different values for J and obtaining the corresponding values for θ.

Positive and Negative Correction.

The most common method of tooth correction for spur gears of full depth proportions, is the one in which the pinion tooth form is displaced outwards, and the wheel tooth inwards by the same amount, which is termed positive and negative tooth correction.

The correction is carried out by simply modifying the outside diameters of the gear blanks, and cutting the teeth with a standard cutter to the standard cutting depth.

Therefore, if the pinion blank is made larger than standard by a certain amount, and the wheel blank smaller by the same amount, it will be found that while the pinion tooth thickness is greater than standard the wheel tooth thickness is also smaller than standard by the same amount. This enables the gears to be meshed together at standard centres, or at centres calculated for spur gears of standard proportions.

The amount of correction necessary really depends upon the number of teeth in the pinion, but at the same time care has to be exercised to see that undercut is not introduced in the wheel by excessive correction of the pinion.

The amount of correction for the pinion denoted by K_p is given in the table, Fig. 21, for pinions of 10 teeth to 19 teeth inclusive, and also indicated is the minimum number of teeth in the wheel,

CORRECTION FACTORS FOR SPUR GEARS
BASED ON 1 DIAMETRAL PITCH
20° PRESSURE ANGLE
STANDARD DEPTH

Nº OF TEETH IN PINION	CORRECTION FACTORS		MINIMUM NUMBER OF TEETH IN MATING WHEEL HAVING CORRECTION FACTOR EQUAL TO K_W	B_W (WHEEL)	$x - \frac{D}{2}\sin^2\psi$ x MUST BE EQUAL TO OR GREATER THAN B_W TO ENSURE FREEDOM FROM UNDERCUT IN WHEEL
	PINION K_P	WHEEL K_W			
19	0·04	− 0·04	19	1·04	1·10
18	0·08	− 0·08	19	1·08	1·10
17	0·12	− 0·12	20	1·12	1·16
16	0·16	− 0·16	20	1·16	1·16
15	0·20	− 0·20	21	1·20	1·22
14	0·24	− 0·24	22	1·24	1·27
13	0·28	− 0·28	23	1·28	1·33
12	0·32	− 0·32	23	1·32	1·33
11	0·36	− 0·36	24	1·36	1·39
10	0·40	− 0·40	25	1·40	1·45

THE CORRECTION FACTORS GIVEN ABOVE SHOULD NOT BE EXCEEDED IN ORDER TO ENSURE THAT A REASONABLE POINT WIDTH IS MAINTAINED.
THEY CAN BE REDUCED HOWEVER IF NECESSARY, BUT SHOULD NOT BE LESS THAN THE VALUES GIVEN BELOW IF ABSOLUTE FREEDOM FROM UNDERCUT IS TO BE MAINTAINED

Nº OF TEETH IN PINION	10	11	12	13	14	15	16	17
CORRECTION FACTOR IN PINION K_P	0·40	0·36	0·30	0·24	0·18	0·12	0·06	0 00

FIG 21. CORRECTION FACTORS FOR SPUR GEARS.

which it is considered should be allowed to mesh with the pinion having the same correction.

The figures given are based on teeth of 1 diametral pitch 20° pressure angle, of standard full depth proportions, and they should not be exceeded if a reasonable point width of tooth is to be maintained.

They can be reduced slightly if necessary but should not be less than the minimum values which are also given, if absolute freedom from undercut is desired.

For other ratios of 20° pressure angle not covered by this table, it will be necessary to consider them under the heading of [illegible] centre distance gearing, which will be described later.

The length of contact, and the point width of the teeth should also be checked, either by layout or calculation, in the same manner as that previously described.

To make the calculations for a pair of gears corrected in this manner, it is necessary to calculate the blank diameters, addendum, dedendum, and circular tooth thickness after which the same procedure can be followed as for a standard gear,

then let Z_1 = number of teeth in wheel.
Z_2 = number of teeth in pinion.
DP = diametral pitch.
D = pitch circle diameter of wheel.
d = pitch circle diameter of pinion.
ψ = pressure angle.
K_p = pinion correction factor.
K_w = wheel correction factor.
A = wheel addendum.
a = pinion addendum.
B = wheel dedendum.
b = pinion dedendum.
t_w = circular thickness of wheel tooth.
t_p = circular thickness of pinion tooth.
J = outside diameter of wheel.
i = outside diameter of pinion.

Wheel.

$$D = \frac{Z_1}{DP}$$

$$K_w = -\ K_p$$

$$A = \frac{1}{DP} - K_p$$

$$J = D + 2A$$

$$B = \frac{1{\cdot}157}{DP} \text{ or } \frac{1{\cdot}250}{DP} + K_p$$

$$t_w = \frac{\pi}{2\,DP} - 2\,(K_w \tan \psi)$$

Pinion.

$$d = \frac{Z_2}{DP}$$

$$K_p = \frac{\text{value from table}}{DP}$$

$$a = \frac{1}{DP} + K_p$$

$$j = d + 2a$$

$$b = \frac{1{\cdot}157}{DP} \text{ or } \frac{1{\cdot}250}{DP} - K_p$$

$$t_p = \frac{\pi}{2\,DP} + 2\,(K_p \tan \psi)$$

Example.—To make the calculations for a pair of spur gears 8 DP, 20° pressure angle, having 12 teeth in the pinion, and 24 teeth in the wheel, we proceed as follows.

According to the table, Fig. 21, the ratio is within the range, and positive and negative correction can be used then :—

Wheel	**Pinion**
$D = \frac{24}{8} = 3{\cdot}000$	$d = \frac{12}{8} = 1{\cdot}500$
$K_w = -K_p = 0{\cdot}04$	$K_p = \frac{0{\cdot}32}{8} = 0{\cdot}04$
$A = \frac{1}{8} - 0{\cdot}04$ $= 0{\cdot}125 - 0{\cdot}04 = 0{\cdot}085$	$a = \frac{1}{8} + 0{\cdot}04$ $= 0{\cdot}125 + 0{\cdot}04 = 0{\cdot}165$
$J = 3{\cdot}000 + 2 \times 0{\cdot}085$ $= 3{\cdot}170$	$j = 1{\cdot}500 + 2 \times 0{\cdot}165$ $= 1{\cdot}830$
$B = \frac{1{\cdot}250}{8} + 0{\cdot}04$ $= 0{\cdot}156 + 0{\cdot}04$ $= 0{\cdot}196$	$b = \frac{1{\cdot}250}{8} - 0{\cdot}04$ $= 0{\cdot}156 - 0{\cdot}04$ $= 0{\cdot}116$
$t_w = \frac{\pi}{2 \times 8} - 2\,(0{\cdot}04 \tan 20°)$ $= 0{\cdot}1963 - 0{\cdot}029$ $= 0{\cdot}1672$	$t_p = \frac{\pi}{2 \times 8} + 2\,(0{\cdot}04 \tan 20°)$ $= 0{\cdot}1963 + 0{\cdot}029$ $= 0{\cdot}2254$

To check these values $(t_w + t_p)$ should be equal to $\frac{\pi}{DP}$ and in example

$$0{\cdot}1672 + 0{\cdot}2254 = 0{\cdot}3926$$

To check for length of contact and referring to Fig. 22, the contact ratio is equal to 1·42 which is satisfactory for obtaining continuity of contact when

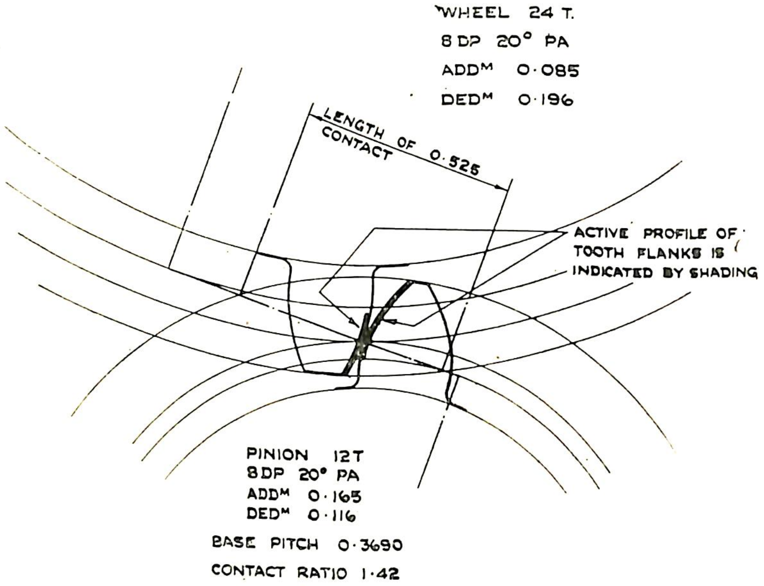


FIG. 22. LAYOUT TO CHECK CONTACT RATIO OF 12 AND 24 TOOTH SPUR GEARS.

$$\text{Contact ratio} = \frac{\text{Length of Contact}}{\text{Base Pitch}} = \frac{0{\cdot}525}{0{\cdot}369}$$

$$\text{and Base Pitch} = \frac{\pi}{\text{DP}} \cos \psi = \frac{\pi}{8} \cos 20^\circ = 0{\cdot}369$$

Although the contact ratio is satisfactory, undercutting of the tooth flanks is not present, and the point width of the pinion tooth is also suitable, it should be noted from the figure that the active profile of the pinion tooth flank extends very near the base circle diameter.

This condition should be avoided wherever possible; therefore pinions having very small numbers of teeth should be used only in cases of absolute necessity.

Extended Centre Distance Gearing.

One great advantage of using the involute form of tooth for spur gears is that the gear centres can be increased within reasonable proportions without affecting the velocity ratio of the two gears.

Should the centres of a pair of standard gears be increased, however, it would be found that excessive backlash had been introduced, and it is then necessary to increase the blank diameters and tooth thickness of both wheel and pinion to overcome the deficiency.

Due however to the fact that standard cutters can be used to cut extended centre distance gearing, and both gears are positively corrected, the sum of the total correction will be found to be slightly greater than the actual increase in centre distance. It will also be found that the greater the increase in centre distance the greater the reduction in the total depth of tooth from standard, providing that the standard clearance is maintained.

One great disadvantage of the system is that the centre distance is not standard, in fact the gears are designed around the centres desired, but on the other hand, standard generating cutters can be used in practically every case although this is done by introducing the module system of cutter standardisation.

Where the system is found very useful is when a pair of spur gears are required to mesh at centres which are not standard, a condition often experienced in machine tool design.

It is then possible, by using the formula given below, to calculate particulars of a pair of gears which will run together quite satisfactorily, and in fact better than if the gears had been meshing at standard centres.

This is due to the fact that full advantage is taken of the maximum rolling conditions available, for increased rolling as distinct from sliding gives increased efficiency and lengthens the life of the gears. It is agreed that more lengthy calculations are necessary, but the results obtained fully justify the extra work involved.

To illustrate the improved rolling conditions obtained by increasing the centre distance, diagrams are included as follows :—

Fig. 23—25/30 ratio 10 DP, $14\frac{1}{2}$° pressure angle, standard proportions.
Fig. 24—25/30 ratio 10 DP, 20° pressure angle, standard proportions.
Fig. 25—25/30 ratio cut with standard 10 DP, $14\frac{1}{2}$ pressure angle cutter, but gearing at extended centres.

It will be noted on each figure that the amount of rolling in relation to the amount of sliding has been plotted, and in the case of the gears running at extended centres, rolling is considerably in excess of sliding.

A cutter pressure angle of $14\frac{1}{2}$° is used for extended centre distance gearing as it has been found more suitable for this method of correction than 20° ; therefore any ratio above 40/40, *i.e.*, a pinion having more than 40 teeth gearing with a wheel having more than 40 teeth, need not be considered.

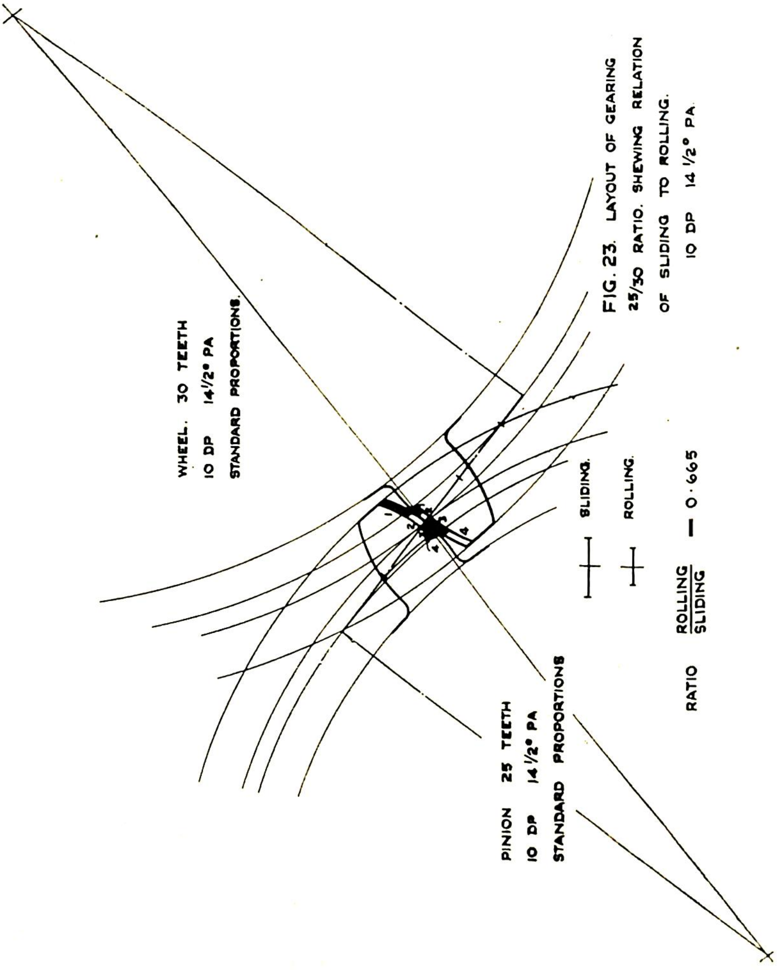


FIG. 23. LAYOUT OF GEARING 25/30 RATIO. SHEWING RELATION OF SLIDING TO ROLLING. 10 DP 14½° PA.

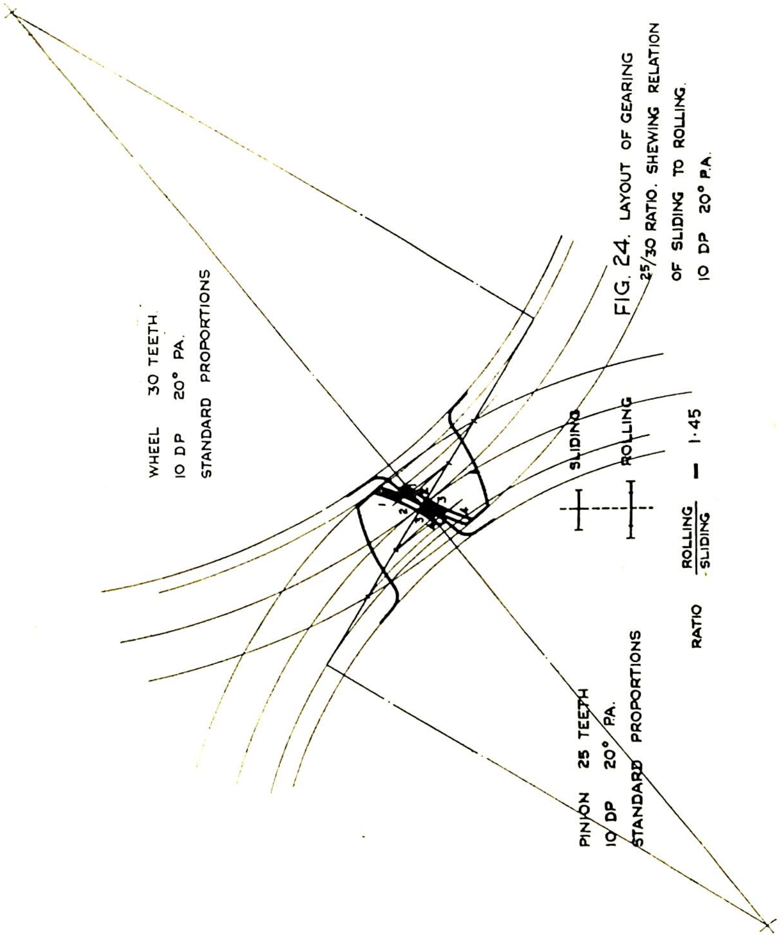


FIG. 24. LAYOUT OF GEARING 25/30 RATIO. SHEWING RELATION OF SLIDING TO ROLLING. 10 DP 20° P.A.

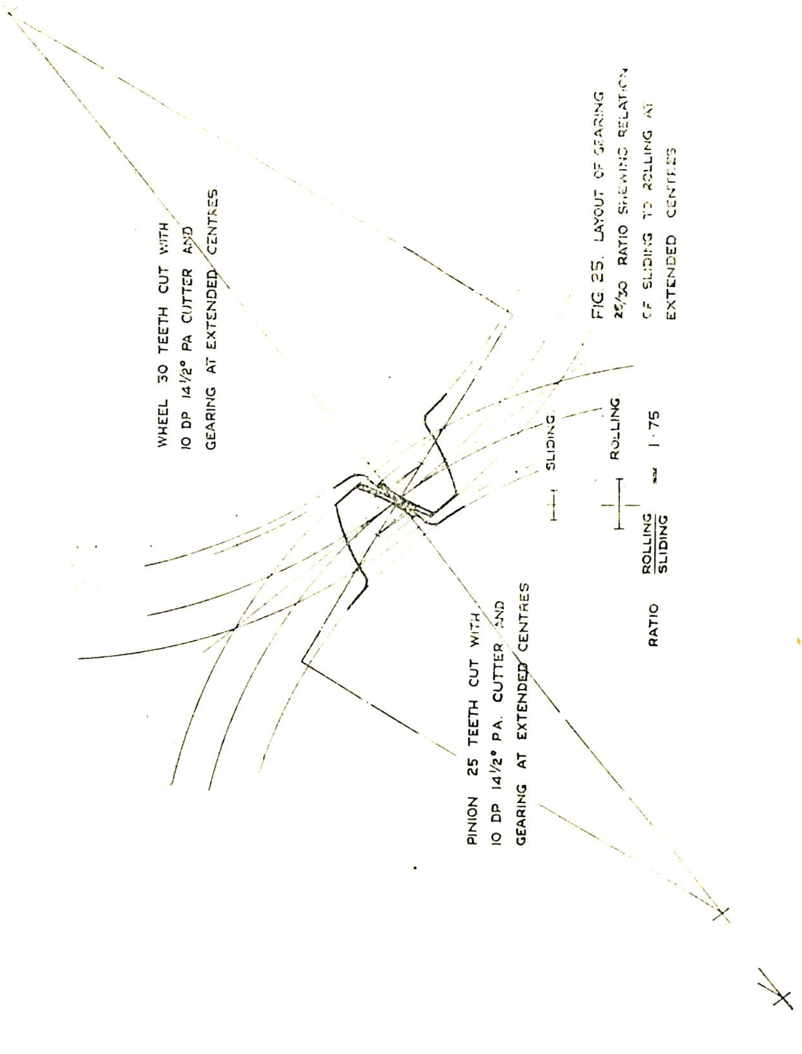

WHEEL 30 TEETH CUT WITH
10 DP 14½° PA CUTTER AND
GEARING AT EXTENDED CENTRES.
PINION 25 TEETH CUT WITH
10 DP 14½° P.A. CUTTER AND
GEARING AT EXTENDED CENTRES
SLIDING.
ROLLING
RATIO $\frac{\text{ROLLING}}{\text{SLIDING}}$ = 1·75

FIG. 25. LAYOUT OF GEARING 25/30 RATIO SHEWING RELATION OF SLIDING TO ROLLING AT EXTENDED CENTRES

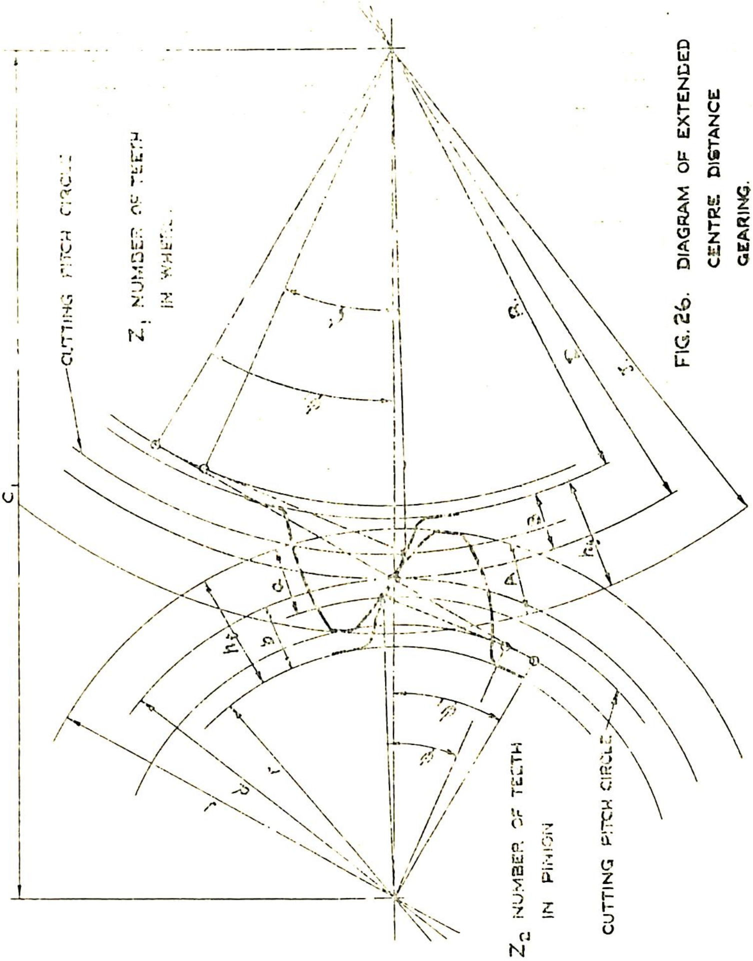


FIG. 26. DIAGRAM OF EXTENDED CENTRE DISTANCE GEARING.

The reason for this is that very little improvement would be obtained by increasing the centres above this range, and the gears could be made either 14½° or 20° pressure angle, of standard proportions and to run at standard centres.

The only reason therefore for increasing the centres above the range specified would be when it was desired to run the gears at extended centres for the sole purpose of accommodating the design.

The calculations are based on the number of teeth in the wheel and pinion, the working centre distance, and the pitch of the cutter used to cut the gears, and referring to Fig. 26 and Fig. 27 we have :

C_1 = working centre distance.
C = basic centre distance.
Z_1 = number of teeth in wheel.
Z_2 = number of teeth in pinion.
M = module pitch of gears.
M_c = module pitch of cutter.
ψ = pressure angle of cutter.
ψ_1 = working pressure angle of gears.
B = dedendum of wheel } based on working
b = dedendum of pinion. } pitch diameters.

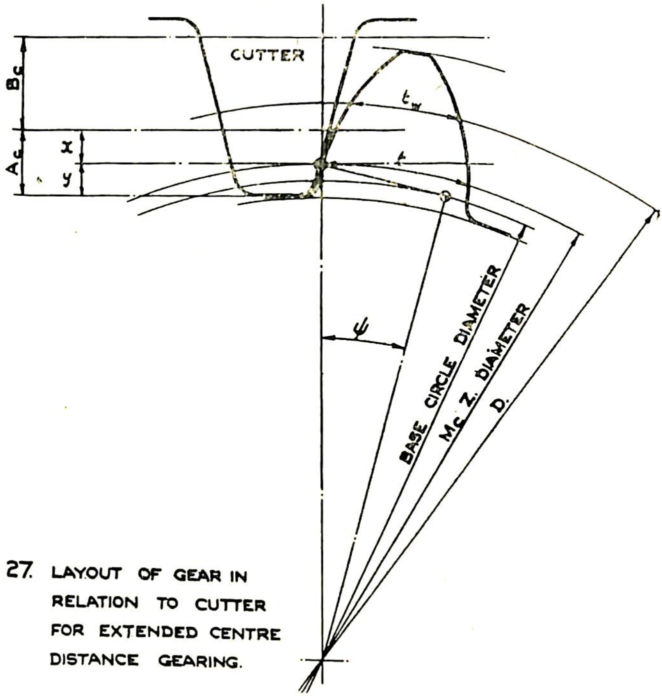


FIG. 27. LAYOUT OF GEAR IN RELATION TO CUTTER FOR EXTENDED CENTRE DISTANCE GEARING.

A_c = addendum of cutter.
B_c = dedendum of cutter.
A = addendum of wheel } based on cutting
a = addendum of pinion } pitch diameter.
D = working pitch diameter of wheel.
d = working pitch diameter of pinion.
J = outside diameter of wheel.
j = outside diameter of pinion.
R = root diameter of wheel.
r = root diameter of pinion.
h_t = full depth of tooth.
x_1 = cutter withdrawal on wheel.
x_2 = cutter withdrawal on pinion.
y_1 = dedendum of wheel } based on cutting
y_2 = dedendum of pinion. } pitch diameter.
t_w = circular tooth thickness of wheel } measured on cutting pitch diameter.
t_p = circular tooth thickness of pinion }
k = constant for working clearance.
0·157 for hobbing or rack cutting.
0·250 for gear shaping.
t_{ww} = circular tooth thickness of wheel } measured on working pitch diameter.
t_{wp} = circular tooth thickness of pinion }

Then given Z_1, Z_2 and C_1

$$M = \frac{2\,C_1}{Z_1 + Z_2}$$

Let M_c = standard module of cutter.
ψ = pressure angle of cutter.

$$\text{then } C = \frac{M_c\,(Z_1 + Z_2)}{2}$$

$$\cos\psi_1 = \frac{C\cos\psi}{C_1}$$

$$(x_1 + x_2) = \frac{C\,(\text{inv. }\psi_1 - \text{inv. }\psi)}{\tan\psi}$$

$$\frac{R + r}{2} = (C - 2A_c) + (x_1 + x_2)$$

$$B = \frac{C_1 - (R + r)/2}{1 + \sqrt{Z_2/Z_1}}$$

$$b = \frac{C_1 - (R + r)/2}{1 + \sqrt{Z_1/Z_2}}$$

$$\begin{aligned} D &= MZ_1 & \text{and} \quad d &= MZ_2 \\ R &= D - 2B & r &= d - 2b \\ J &= R + 2h_t & j &= r + 2h_t \end{aligned}$$

$$\text{when } h_t = C_1 - kM_c - \frac{(R + r)}{2}$$

To calculate addendum and circular thickness of tooth on cutting pitch diameter for wheel and pinion we have :—

$$y_1 = \frac{M_c Z_1 - R}{2} \quad \text{and} \quad y_2 = \frac{M_c Z_2 - r}{2}$$

$$x_1 = A_c - y_1 \quad \text{and} \quad x_2 = A_c - y_2$$

$$t_w = \frac{M_c \pi}{2} + 2\,(x_1 \tan \psi) \text{ and } t_p = \frac{M_c \pi}{2} + 2\,(x_2 \tan \psi)$$

$$A = \frac{J - M_c Z_1}{2} \quad \text{and} \quad a = \frac{j - M_c Z_2}{2}$$

The values obtained for A, a, t_w and t_p can then be corrected for vernier settings in the same manner as for a standard gear, and an allowance for backlash made.

To check the above calculations, the circular tooth thickness of both wheel and pinion are calculated on the working pitch circle diameters D and d using t_w and t_p.

The sum of the values obtained should, if the calculations have been made correctly, be equal to M π.

Thus given t_w, t_p, D, d, ψ and ψ_1, we have

$$t_{ww} = D\left(\frac{t_w}{M_c Z_1} + \text{inv. } \psi - \text{inv. } \psi_1\right) \text{ for the wheel}$$

$$t_{wp} = d\left(\frac{t_p}{M_c Z_2} + \text{inv. } \psi - \text{inv. } \psi_1\right) \text{ for the pinion}$$

$$\text{then } (t_{ww} + t_{wp}) = M\,\pi.$$

The pressure angle ψ_1 really controls the design of the gear teeth for extended centre distance gearing, but this in turn is also controlled by the pitch or module of the generating cutter available.

For example the gear teeth may be set out to the very best advantage, and the value ψ_1 then modified, to the possible detriment of the running conditions in order to use a standard cutter.

If, however, certain intermediate cutter standards are introduced in the module system, it will be found that it is only necessary to make a slight departure from the best running conditions.

Example.—Calculate particulars of a pair of spur gears to mesh at 2·850 centres 25/30 ratio using a 10 DP 14½° PA cutter.

$$\text{Then } M = \frac{2 \times 2{\cdot}850}{(30+25)} = 0{\cdot}10363$$

$$\text{and } M_c = 0{\cdot}100 \text{ or module for 10 DP}$$

$$\psi = 14^\circ - 30'$$

$$\text{then } C = \frac{0{\cdot}100\ (30 + 25)}{2} = 2{\cdot}750$$

$$\cos \psi_1 = \frac{2{\cdot}750 \times 0{\cdot}96815}{2{\cdot}850} = 0{\cdot}93418$$

$$= 20^\circ - 54'$$

$$(x_1 + x_2) = \frac{2{\cdot}750\ (0{\cdot}017089 - 0{\cdot}0055448)}{0{\cdot}25862}$$

$$= 0{\cdot}1227$$

$$\frac{(R+r)}{2} = (2{\cdot}750 - 2 \times 0{\cdot}125) + 0{\cdot}1227$$

$$= 2{\cdot}6227$$

$$B = \frac{2{\cdot}850 - 2{\cdot}6227}{1 + \sqrt{25/30}} \qquad b = \frac{2{\cdot}850 - 2{\cdot}6227}{1 + \sqrt{30/25}}$$

$$= \frac{0{\cdot}2273}{1{\cdot}91287} \qquad = \frac{0{\cdot}2273}{2{\cdot}0954}$$

$$= 0{\cdot}1188 \qquad = 0{\cdot}1084$$

$$D = 0{\cdot}10363 \times 30 \qquad d = 0{\cdot}10363 \times 25$$

$$= 3{\cdot}1089 \qquad = 2{\cdot}5907$$

$$R = 3{\cdot}1089 - 2 \times 0{\cdot}1188 \qquad r = 2{\cdot}5907 - 2 \times 0{\cdot}1084$$

$$= 2{\cdot}8713 \qquad = 2{\cdot}3739$$

$$h_t = 2{\cdot}850 - 0{\cdot}250 \times 0{\cdot}100 - 2{\cdot}6227$$

$$= 0{\cdot}2023$$

$$J = 2{\cdot}8713 + 2 \times 0{\cdot}2023 \qquad j = 2{\cdot}3739 + 2 \times 0{\cdot}2023$$

$$= 3{\cdot}2759 \qquad = 2{\cdot}7785$$

$$y_1 = \frac{0{\cdot}100 \times 30 - 2{\cdot}8713}{2} \qquad y_2 = \frac{0{\cdot}100 \times 25 - 2{\cdot}3739}{2}$$

$$= 0{\cdot}0643 \qquad = 0{\cdot}0630$$

$$x_1 = 0{\cdot}125 - 0{\cdot}0643 \qquad x_2 = 0{\cdot}125 - 0{\cdot}0630$$

$$= 0{\cdot}0607 \qquad = 0{\cdot}062$$

$$t_w = \frac{0{\cdot}100 \times \pi}{2} + 2\ (0{\cdot}0607 \tan 14\tfrac{1}{2}^\circ) \qquad t_p = \frac{0{\cdot}100 \times \pi}{2}$$

$$+ 2\ (0{\cdot}062 \times \tan 14\tfrac{1}{2}^\circ)$$

$$= 0{\cdot}1884 \qquad = 0{\cdot}1891$$

$$A = \frac{3\cdot2759 - 3\cdot000}{2} \qquad a = \frac{2\cdot7785 - 2\cdot500}{2}$$

$$= 0\cdot1379 \qquad = 0\cdot1392$$

$$t_{ww} = 3\cdot1089 \left(\frac{0\cdot1884}{3\cdot000} + 0\cdot0055448 - 0\cdot017089\right)$$

$$= 0\cdot15936$$

$$t_{wp} = 2\cdot5907 \left(\frac{0\cdot1891}{2\cdot500} + 0\cdot0055448 - 0\cdot017089\right)$$

$$= 0\cdot16605$$

$$\text{and } (0\cdot15936 + 0\cdot16605) = 0\cdot10363 \times \pi$$

$$= 0\cdot32541 = 0\cdot32556$$

which for all practical purposes is the same.

The formulae previously given refer only of course to a pair of gears, gearing at a given centre distance.

Where it is desired, however, to mesh three or more gears together, all gearing on extended centres, further calculations are necessary for the additional gears in the train.

This problem may also arise in the case of a gear train of three or more gears having two gears with positive and negative tooth correction, especially if it is required to mesh a wheel with say the first driven wheel which has negative correction.

The difficulty can be overcome in both cases however in two ways, the first which is by far the easiest, being to mount say a third gear on the same shaft as the driven wheel, and use this gear to drive a fourth gear on the third driven shaft.

In this way the two pairs of gears can be treated independently of each other, and arranged to gear at the centres desired.

The second method is more involved, as in this instance it is being assumed that three gears have to run in line, and the particulars of the first two gears Z_2 and Z_1 have been calculated and gear Z_3 must be made to run with gear Z_1.

In any event gear Z_3 cannot be made to run at standard centres with gear Z_1 which has been corrected, unless gear Z_3 has the same number of teeth as gear Z_2, and therefore has the same correction.

To avoid the extra work involved, it is advisable, wherever possible, when such gear trains are required, to arrange the numbers of teeth to be such that the minimum number of teeth in the smallest gear in the train is not less than 17 for full depth proportions, or 14 teeth using the stub tooth standard.

If smoother running is necessary, however, and a high class job is required, the particulars of the third gear in the train can be calculated as follows, although it is always advisable to deal first with the gear pair having the smallest pinion :—

let Z_1 = second wheel in train.
Z_2 = first pinion in train.
Z_3 = third gear in train.

using the same notation as before and using the same module of cutter for all gears.

$$C = \frac{M_c (Z_1 + Z_3)}{2}$$

$$\text{and } (x_1 + x_3) = \frac{C (\text{inv. } \psi_1 - \text{inv. } \psi)}{\tan \psi}$$

To determine the value of ψ_1, it is necessary to layout the gear teeth and ascertain the best running conditions of Z_1 with Z_3.

Then $x_3 = (x_1 + x_3) - x_1$ taking the value of x_1 from the previously calculated gear pair.

$$\text{and } R_3 = (M_c Z_3 - 2A_c) + 2x_3$$

$$t_3 = \frac{M_c \pi}{2} + 2 (x_3 \tan \psi)$$

$$C_1 = \frac{(R + R_3)}{2} + h_t + kM_c$$

$$M = \frac{2C_1}{(Z_1 + Z_3)} \text{ then } D_3 = MZ_3$$

$$\text{and } D = MZ_1$$

To prove the above calculations, the circular tooth thickness of both Z_1 and Z_3 measured on the working pitch circle diameters D and D_3, are obtained as follows :—

$$t_{ww} = D \left[\frac{t_w}{M_c Z_1} + \text{inv. } \psi - \text{inv. } \psi_1 \right]$$

when value for t_w is obtained from previous calculations.

$$t_{w3} = M \pi - t_{ww} \qquad (1)$$

$$\text{or } t_{w3} = D_3 \left[\frac{t_3}{M_c Z_3} + \text{inv. } \psi - \text{inv. } \psi_1 \right] \qquad (2)$$

If the calculations have been made correctly then the results at (1) and (2) should be equal, then to obtain outside diameter J_3 of gear Z_3.

$$J_3 = R_3 + 2h_t$$

An example is now given, showing how the particulars of a third gear having 40 teeth, meshing with the two gears in the previous example, are calculated.

$$\begin{aligned} \text{then } Z_1 &= 30 \\ Z_2 &= 25 \\ Z_3 &= 40 \\ C &= \frac{0{\cdot}100\ (30 + 40)}{2} \\ &= 3{\cdot}500. \end{aligned}$$

Obtain value for ψ_1 from layout showing the best running condition. In any case the angle selected should be less than 20° - 54′ as 40 is greater than 30 teeth, and in this example 18° is considered suitable.

$$\begin{aligned} \text{then } (x_1 + x_3) &= \frac{3{\cdot}500\ (\text{inv. } 18^\circ - \text{inv. } 14\tfrac{1}{2}^\circ)}{\tan 14\tfrac{1}{2}^\circ} \\ &= \frac{3{\cdot}500\ (0{\cdot}010760 - 0{\cdot}0055448)}{0{\cdot}25862} \\ &= 0{\cdot}0705. \end{aligned}$$

taking the value of x_1 as 0·0607 from the previous example

$$\begin{aligned} x_3 &= 0{\cdot}0705 - 0{\cdot}0607 \\ &= 0{\cdot}0098 \\ R_3 &= (0{\cdot}100 \times 40 - 2 \times 0{\cdot}125) + 2 \times 0{\cdot}0098 \\ &= 3{\cdot}7696 \\ t_3 &= \frac{0{\cdot}100 \times \pi}{2} + 2\ (0{\cdot}0098 \tan 14\tfrac{1}{2}^\circ) \\ &= 0{\cdot}15708 + 0{\cdot}00506 \\ &= 0{\cdot}16214 \end{aligned}$$

taking the value of "R" and "h_t" from the previous example

$$\begin{aligned} C_1 &= \frac{(2{\cdot}8713 + 3{\cdot}7696)}{2} + 0{\cdot}2023 + 0{\cdot}250 \times 0{\cdot}100 \\ &= 3{\cdot}5427 \\ \text{then } M &= \frac{2 \times 3{\cdot}5427}{(30 + 40)} = 0{\cdot}10122 \\ D_3 &= 0{\cdot}10122 \times 40 = 4{\cdot}0488 \\ \text{and } D &= 0{\cdot}10122 \times 30 = 3{\cdot}0366 \end{aligned}$$

To prove the calculations, and taking the value for t_w for Z^1 from the previous calculations.

$$t_{ww} = 3{\cdot}0366 \left[\frac{0{\cdot}1884}{0{\cdot}100 \times 30} + \text{inv. } 14\tfrac{1}{2}^\circ - \text{inv.} 18^\circ\right]$$

(on 3·0366 dia.)

$$\begin{aligned} &= 3{\cdot}0366\ [0{\cdot}0628 + 0{\cdot}0055448 - 0{\cdot}010760] \\ &= 0{\cdot}1748 \\ t_{w3} &= 0{\cdot}10122 \times \pi - 0{\cdot}1748 \\ &= 0{\cdot}31799 - 0{\cdot}1748 \\ &= 0{\cdot}1431 \end{aligned}$$

$$\begin{aligned} t_{w3} &= 4{\cdot}0488\ \frac{0{\cdot}16214}{0{\cdot}100 \times 40} + \text{inv. } 14\tfrac{1}{2}^\circ - \text{inv. } 18^\circ \\ &= 0{\cdot}1430 \end{aligned}$$

the two results are the same within 0·0001, therefore the calculations have been made correctly.

$$\begin{aligned} J_3 &= 3{\cdot}7696 + 2 \times 0{\cdot}2023 \\ &= 4{\cdot}1742 \end{aligned}$$

A layout of the three gears is shown in Fig. 28.

Involute Tooth Action.

Pure rolling of spur, helical and bevel gears having involute tooth form, occurs only on the pitch circle diameters.

For this reason it is impossible to eliminate sliding altogether, but by modifying the tooth form it has been shown that the amount of sliding can be reduced considerably.

A good rule to remember is that the active profile of an involute tooth should not be taken to, or very near the base circle diameter if smooth tooth action is to be obtained.

Although in the previous examples sliding and rolling have been plotted on the diagrams, Figs. 23, 24 and 25, the amount of specific sliding can be calculated and charted as shown in Figs. 29 and 30.

For smooth running the specific sliding value should be as low as possible, while the curvature of the lines shown on the small graphs should be about equal for both wheel and pinion.

If a sudden change is shown on the curve it will indicate that excessive wear will take place at the point indicated and a smooth curve with little change should be the standard of good gearing. For this reason two examples are shown, the first, Fig. 29, depicting a pair of spur gears 12/25 ratio $14\frac{1}{2}^\circ$ pressure angle gearing at standard centres, while in Fig. 30 the same gears are shown gearing at extended centres. From the layouts, calculations are made and the amount of specific sliding obtained in the roots and at the top of both wheel and pinion.

Then at (pinion root) GE $= b_1$ HE $= b_2$
(wheel top)
at (pinion top) GF $= b_1$ HF $= b_2$
(wheel root)

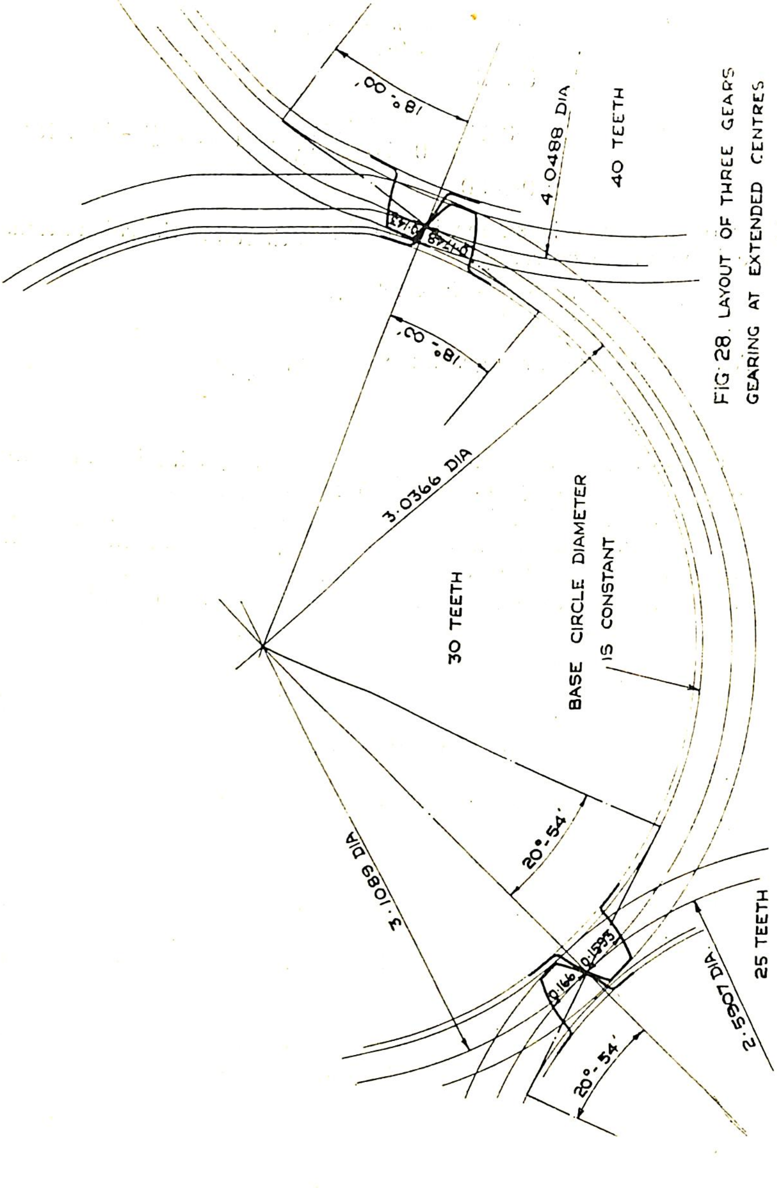


FIG 28. LAYOUT OF THREE GEARS GEARING AT EXTENDED CENTRES

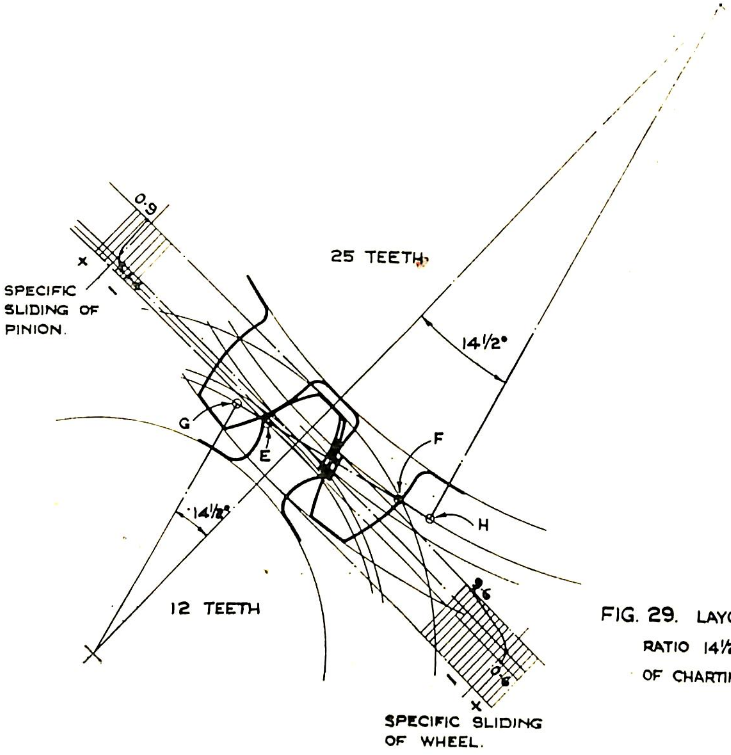


FIG. 29. LAYOUT OF GEARS 12/25 RATIO 14½° P.A. SHEWING METHOD OF CHARTING SPECIFIC SLIDING

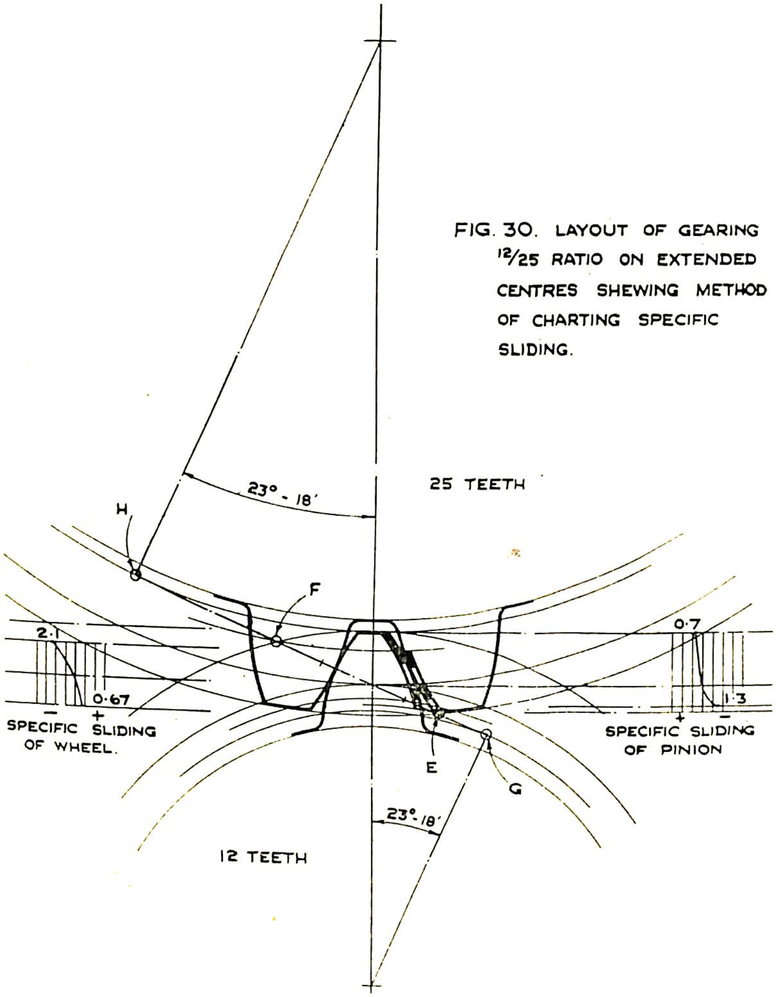


FIG. 30. LAYOUT OF GEARING 12/25 RATIO ON EXTENDED CENTRES SHEWING METHOD OF CHARTING SPECIFIC SLIDING.

Specific sliding driving gear (pinion)

$$= \frac{b_1 Z_1 - b_2 Z_2}{b_1 Z_1}$$

Driven gear (wheel)

$$= \frac{b_2 Z_2 - b_1 Z_1}{b_2 Z_2}$$

when Z_1 = number of teeth in wheel
Z_2 = number of teeth in pinion

In example, Fig. 29, and scaling diagram using millimetres for ease of measurement

Specific sliding with pinion driving

$$= \frac{9 \times 25 - 49 \times 12}{9 \times 25} = -\ 1{\cdot}4 \text{ at root}$$

$$= \frac{48{\cdot}5 \times 25 - 9{\cdot}5 \times 12}{48{\cdot}5 \times 25} = +\ 0{\cdot}9 \text{ at top}$$

with wheel driving

$$= \frac{9{\cdot}5 \times 12 - 48{\cdot}5 \times 25}{9{\cdot}5 \times 12} = -\ 9{\cdot}6 \text{ at root}$$

$$= \frac{49 \times 12 - 9 \times 25}{49 \times 12} = +\ 0{\cdot}6 \text{ at top}$$

and for example, Fig. 30, with pinion driving

$$= \frac{13{\cdot}5 \times 25 - 85 \times 12}{13{\cdot}5 \times 25} = -\ 1{\cdot}3 \text{ at root}$$

$$= \frac{59 \times 25 - 39 \times 12}{59 \times 25} = +\ 0{\cdot}7 \text{ at top}$$

with wheel driving

$$= \frac{39 \times 12 - 59 \times 25}{39 \times 12} = -\ 2{\cdot}1 \text{ at root}$$

$$= \frac{85 \times 12 - 13{\cdot}5 \times 25}{85 \times 12} = +\ 0{\cdot}67 \text{ at top}$$

From Fig. 29 it will be noted that considerable wear will take place on the wheel flanks due to the sudden change in curvature of the wheel curve, while less wear will take place on the pinion flanks.

On the other hand the gear pair shown in Fig. 30, gearing at extended centres, show considerable improvement on the gear pair shown in Fig. 29.

The specific sliding is about equal in the roots of both wheel and pinion, while the difference at the tops is only slight.

It should also be noted that in the case of the wheel curves, there is a vast difference between the two, thus indicating that the gear pair shown in Fig. 30 will run smoother and with less wear taking place on the tooth flanks.

Before finalising the design, however, it is always advisable to check the contact ratio and point width of the gears, either by layout or calculation.

This precaution is very necessary, as the centres may be increased to such an extent that the contact ratio is made less than 1·000, whereas for safety a minimum of 1·15 is advised if continuous contact is to be obtained.

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24. ¾" Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞" ,, ,, ,, (30 ton yield).
26. 1" ,, ,, ,, (30 ton yield).
27. ¾" ,, ,, ,, (40 ton yield).
28. ⅞" ,, ,, ,, (40 ton yield).
29. 1" ,, ,, ,, (40 ton yield).
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66. Ring with Uniform Internal Load (Tangential Stress)
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
86. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
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